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Complete Syllabus • Conceptual Flowcharts • Comparative Tables • Solved Practice Questions

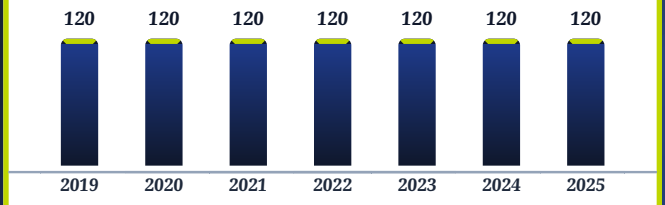
# COMPLETE HIGHER MATHEMATICS — NDA, AIR FORCE X, NAVY, CUET & PGT

Sets, Relations & Functions, Complex Numbers, Quadratic Equations, Permutations & Combinations, Binomial Theorem, Matrices & Determinants, Trigonometry, 2D Conic Sections, Differential & Integral Calculus, Vectors, 3D Geometry & Probability

### Topic-Wise Exam Weightage Distribution (%)




### Previous Years' Question Trend Analysis




**Target Competitive, Entrance & Departmental Examinations**

**UPSC NDA & NA (120 Questions / 300 Marks) • Air Force Group-X • Navy SSR • Coast Guard • CUET UG Maths • TGT/ PGT Mathematics**

## Core Syllabus Pillars & Exam Question Weightage (%)

- Differential & Integral Calculus, Limits, Maxima-Minima & Diff. Equations (28%)
- Algebra (Sets, Functions, Complex Numbers, Quadratics, P&C, Binomial, AP/GP) (24%)
- Matrices & Determinants (Inverse, Adjoint, Cramer's Rule, Eigenvalues) (16%)
- 2D Coordinate Geometry (Lines, Circle, Parabola, Ellipse, Hyperbola) & Trig (18%)
- Vector Algebra, 3D Analytical Geometry, Statistics & Bayes' Probability (14%)

**Complete Theory**
NCERT & Official Sources

**Visual Flowcharts**
Comparative Matrices

**Memory Tricks**
Exam Trap Alerts

**75+ Solved MCQs**
One-Liner Capsules

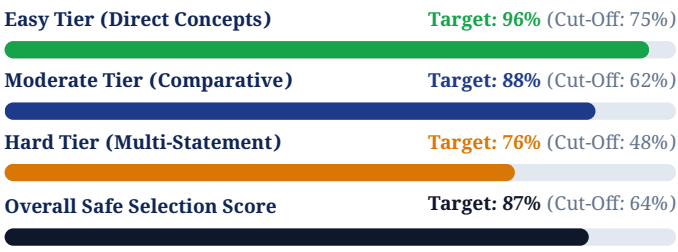
# Preface, Exam Weightage Dashboard & Study Strategy — COMPLETE HIGHER MATHEMATICS — NDA, AIR FORCE X, NAVY, CUET & PGT

IUC Academy (Inclusive Universal Careers Academy | www.iucacademy.com) • Official English Medium Study Notes

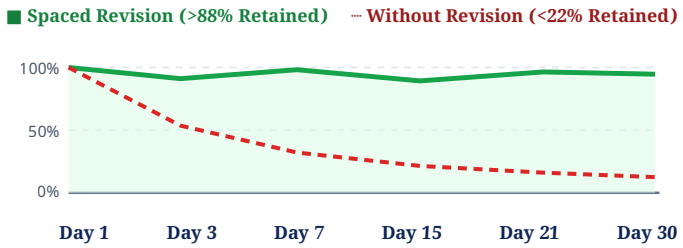
## Dear Aspirants,

Modern competitive, entrance, and departmental promotion examinations demand a combination of deep conceptual clarity, statutory precision, and rapid revision tools. Prepared by **Inclusive Universal Careers Academy (IUC Academy — www.iucacademy.com)**, these comprehensive English-medium study notes integrate **structured chapter theory, 4-node conceptual process flowcharts, multi-column comparative tables, inline mathematical/scientific formulas, 'Did You Know?' boxes, 'Exam Trap Alert' callouts, 'Memory Trick' mnemonics, Chapter-End Analytical Synthesis Matrices, 75+ Step-by-Step Solved MCQs, and Rapid-Fire One-Liner Revision Capsules**.

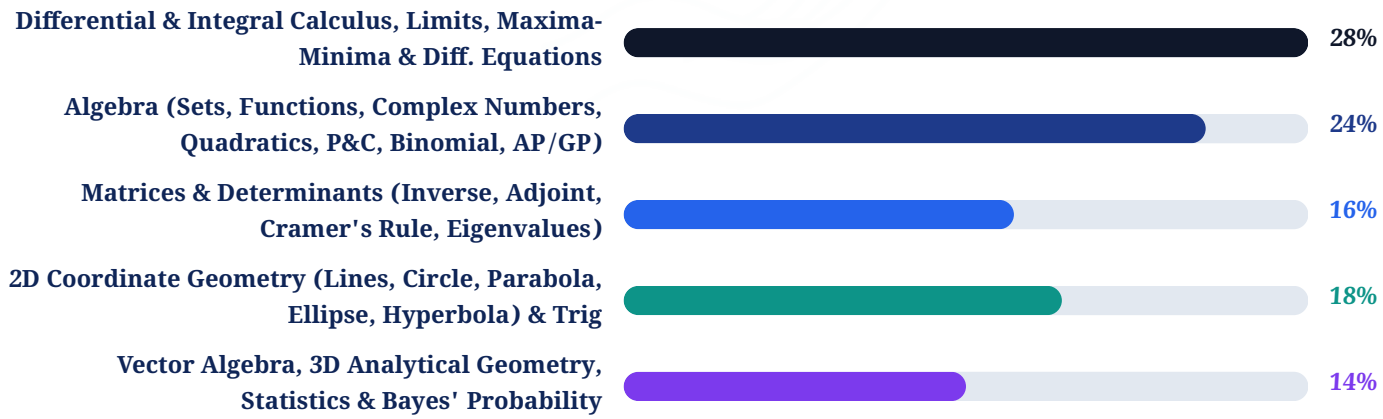
### Target Accuracy vs Qualifying Cut-Off Benchmark (%)




### Spaced Revision (1-3-7-21 Day Rule) & Retention Curve




### Topic-Wise Exam Question Weightage Index (%)




### Verified Official Curriculum & Statutory Reference Portals

- UPSC Official Portal (https://upsc.gov.in/)
- NCERT Class 11 & 12 Mathematics (https://ncert.nic.in/textbook.php)
- IUC Academy Official Portal (https://www.iucacademy.com)

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## 11. Rapid-Fire Last-Minute One-Liner Revision Capsule

**Published by:** Inclusive Universal Careers Academy (IUC Academy) | Official Portal: IUC Academy Official Website

**Syllabus & Curriculum Alignment:** Strictly aligned with the Union Public Service Commission (UPSC) NDA Examination Portal and National Council of Educational Research and Training (NCERT) Class XI & XII Textbooks.

# Chapter *1:* Sets, Relations, Functions & Inverse Trigonometric Functions

**Chapter Key Study Pillars — Chapter 1: Sets, Relations, Functions & Inverse Trigonometric Functions**

| Key Study Pillar 1 | Key Study Pillar 2 | Key Study Pillar 3 |
|---|---|---|
| **Fundamental Theory of Sets and Operations** | **Cartesian Product and Binary Relations** | **Functions, Mappings & Inverse Trigonometric Functions** |

## *1.1* Fundamental Theory of Sets and Operations

A set is a well-defined collection of distinct objects, numbers, or mathematical entities. In competitive examinations conducted via UPSC NDA and curriculum frameworks published by NCERT, set theory serves as the foundational axiomatic language for modern algebra, calculus, and probability theory.

• **Representation of Sets:** Sets are represented either in Roster Form (Tabular Form), where elements are listed explicitly within braces such as A = {2, 3, 5, 7}, or in Set-Builder Form (Rule Method), where a characterizing property defines membership, such as A = {x : x is a prime number less than 10}.

• **Cardinality and Subsets:** The cardinal number of a finite set A, denoted as n(A) or |A|, represents the total count of distinct elements in A. If every element of set A is also an element of set B, then A is a subset of B (A subset_of B). For a finite set with n elements, the total number of subsets is 2^n, the total number of proper subsets is (2^n - 1), and the total number of non-empty proper subsets is (2^n - 2).

• **Power Set:** The collection of all possible subsets of a set A is called its Power Set, denoted by P(A). If n(A) = m, then n(P(A)) = 2^m, and n(P(P(A))) = 2^(2^m).

• **Set Operations and Algebraic Laws:**

• **Union of Sets (A union B):** The set of elements belonging to A, or B, or both.

• **Intersection of Sets (A intersect B):** The set of elements common to both A and B. Two sets are disjoint if (A intersect B) = phi (empty set).

• **Difference of Sets (A - B):** The set of elements belonging to A but not to B, equivalent to A intersect B'.

• **Symmetric Difference (A delta B):** Defined as (A - B) union (B - A) or (A union B) - (A intersect B).

• **De Morgan's Laws:** For any two subsets A and B of a universal set U, (A union B)' = A' intersect B' and (A intersect B)' = A' union B'.

• **Inclusion-Exclusion Principle for Cardinality:**

• For two finite sets A and B: n(A union B) = n(A) + n(B) - n(A intersect B).

• For three finite sets A, B, and C: n(A union B union C) = n(A) + n(B) + n(C) - n(A intersect B) - n(B intersect C) - n(C intersect A) + n(A intersect B intersect C).

• Number of elements in exactly one of the three sets: n(A) + n(B) + n(C) - 2n(*A intersect B*) - 2n(B intersect C) - 2n(*C intersect A*) + 3n(A intersect B intersect C).

• Number of elements in exactly two of the three sets: n(A intersect B) + n(B intersect C) + n(C intersect A) - 3*n(A intersect B intersect C).

## *1.2* Cartesian Product and Binary Relations

Given two non-empty sets A and B, the Cartesian product A x B is the set of all ordered pairs (a, b) such that a belongs to A and b belongs to B. If n(A) = p and n(B) = q, then n(A x B) = p  *q, and the total number of possible binary relations from A to B is* 2^(pq).

• **Classification of Relations on a Set A (where n(A) = n):**

• **Reflexive Relation:** A relation R on set A is reflexive if (a, a) belongs to R for every element a in A. Total number of reflexive relations on A is 2^(n*(n - 1)).

• **Symmetric Relation:** A relation R is symmetric if whenever (a, b) belongs to R, then (b, a) also belongs to R. Total number of symmetric relations on A is 2^(n*(n + 1)/2).

• **Anti-Symmetric Relation:** A relation R is anti-symmetric if (a, b) belongs to R and (b, a) belongs to R implies a = b. Total number of anti-symmetric relations is 2^n 3^(*n*(n - 1)/2).

• **Transitive Relation:** A relation R is transitive if (a, b) belongs to R and (b, c) belongs to R implies (a, c) belongs to R.

• **Equivalence Relation:** A relation R on set A that is simultaneously reflexive, symmetric, and transitive. Every equivalence relation partitions the underlying set into mutually disjoint equivalence classes, where the total number of equivalence relations on a set of size n is given by the nth Bell Number B(n) (B(1) = 1, B(2) = 2, B(3) = 5, B(4) = 15, B(5) = 52).

## *1.3* Functions, Mappings & Inverse Trigonometric Functions

A function f from set A to set B (denoted f : A -> B) is a special relation that associates each element x in the domain A with a unique element y = f(x) in the codomain B, as standardized in **NCERT Mathematics Textbooks** and practiced on **IUC Academy**.

• **Domain, Codomain, and Range:** If f : A -> B, then A is the domain, B is the codomain, and the set of all actual image values f(A) = {f(x) : x in A} is the range, where Range subset_of Codomain.

• **Counting Functions (when n(A) = m and n(B) = n):**

• **Total Number of Functions from A to B:** n^m.

• **Injective (One-One) Functions:** Distinct elements of A have distinct images in B. If m <= n, the number of one-one functions is n! / (n - m)! (which is nPm); if m > n, the number of one-one functions is 0.

• **Surjective (Onto) Functions:** Every element in codomain B is the image of at least one element in domain A (Range = Codomain). If m >= n, the count is sum_{r=0 to n} [(-1)^r *nCr* (n - r)^m]; if m < n, the count is 0.

• **Bijective (One-One and Onto) Functions:** A function that is both injective and surjective. Exists only when m = n, and the total count of bijections is n!.

• **Even and Odd Functions:** A function f(x) is even if f(-x) = f(x) (symmetric about the y-axis) and odd if f(-x) = -f(x) (symmetric about the origin). Any function defined on a symmetric domain [-a, a] can be uniquely expressed as the sum of an even function (f(x) + f(-x))/2 and an odd function (f(x) - f(-x))/2.

• **Inverse Trigonometric Functions (ITF) Principal Value Branches & Identities:**

• arcsin(x) + arccos(x) = pi/2 for all x in [-1, 1].

• arctan(x) + arccot(x) = pi/2 for all x in R (all real numbers).

• arcsec(x) + arccsc(x) = pi/2 for all |x| >= 1.

• arctan(x) + arctan(y) = arctan((x + y)/(1 - xy)) *when* xy < 1; and pi + arctan((x + y)/(1 - xy)) *when* x > 0, y > 0, *and* xy > 1.

• 2*arctan*(x) = *arcsin*(2x / (1 + x^2)) for |x| <= 1; 2*arctan*(x) = *arccos*((1 - x^2)/(1 + x^2)) *for* x >= 0; 2arctan(x) = arctan(2*x / (1 - x^2)) for |x| < 1.

**Step-by-Step Conceptual Process Flowchart**




| Function / Concept | Domain | Range / Principal Value Branch | Key Counting / Algebraic Rule | Official Reference |
|---|---|---|---|---|
| **arcsin(x)** | [-1, 1] | [-pi/2, pi/2] | arcsin(-x) = -arcsin(x) (Odd Function) | NCERT Class XII |
| **arccos(x)** | [-1, 1] | [0, pi] | arccos(-x) = pi - arccos(x) (Neither Even nor Odd) | NCERT Class XII |
| **arctan(x)** | R (All Real Numbers) | (-pi/2, pi/2) | arctan(-x) = -arctan(x) (Odd Function) | UPSC NDA Syllabus |
| **arccot(x)** | R (All Real Numbers) | (0, pi) | arccot(-x) = pi - arccot(x) | NCERT Class XII |
| **arcsec(x)** | R - (-1, 1) | [0, pi] - {pi/2} | arcsec(-x) = pi - arcsec(x) | UPSC NDA Syllabus |
| **arccsc(x)** | R - (-1, 1) | [-pi/2, pi/2] - {0} | arccsc(-x) = -arccsc(x) (Odd Function) | IUC Academy Portal |

**Did You Know?:** The empty relation R = phi on a non-empty set A is always symmetric and transitive (vacuously true because the antecedent condition (a, b) in R is never satisfied), but it is never reflexive because (a, a) is missing for elements of the non-empty set A (NCERT).

**Exam Trap Alert:** Candidates frequently assume arcsin(sin(x)) = x for all real x. In reality, arcsin(sin(x)) = x holds ONLY when x lies in the principal branch [-pi/2, pi/2]. For instance, arcsin(sin(2*pi*/3)) *is NOT* 2pi/3; it equals pi - 2*pi/3 = pi/3, which lies inside [-pi/2, pi/2] (UPSC).

★ **Memory Trick:** Group Inverse Trigonometric Functions into Two Families: the **"Sin-Tan-Cosec Family"** has principal ranges centered around 0 ([-pi/2, pi/2] and behaves as odd functions: f(-x) = -f(x)), whereas the **"Cos-Cot-Sec Family"** has principal ranges spanning [0, pi] and satisfies f(-x) = pi - f(x) (IUC Academy).

## Chapter *1* Comprehensive Analytical & Statutory Revision Matrix

| Core Sub-Domain | Key Concept / Provision | Verified Analytical Parameters & Exam Distinctions (UPSC Official Portal) |
|---|---|---|
| **Fundamental Theory of Sets and Operations** | **Representation of Sets** | Sets are represented either in Roster Form (Tabular Form), where elements are listed explicitly within braces such as A = {2, 3, 5, 7}, or in Set-Builder Form (Rule Method), where a characterizing property defines membership, such as A = {x : x is a prime number less than 10}. |
| **Cartesian Product and Binary Relations** | **Cardinality and Subsets** | The cardinal number of a finite set A, denoted as n(A) or \|A\|, represents the total count of distinct elements in A. If every element of set A is also an element of set B, then A is a subset of B (A subset_of B). For a finite set with n elements, the total number of subsets is 2^n, the total number of proper subsets is (2^n - 1), and the total number of non-empty proper subsets is (2^n - 2). |
| **Functions, Mappings & Inverse Trigonometric Functions** | **Power Set** | The collection of all possible subsets of a set A is called its Power Set, denoted by P(A). If n(A) = m, then n(P(A)) = 2^m, and n(P(P(A))) = 2^(2^m). |
| **Fundamental Theory of Sets and Operations** | **Union of Sets (A union B)** | The set of elements belonging to A, or B, or both. |
| **Cartesian Product and Binary Relations** | **Intersection of Sets (A intersect B)** | The set of elements common to both A and B. Two sets are disjoint if (A intersect B) = phi (empty set). |
| **Functions, Mappings & Inverse Trigonometric Functions** | **Difference of Sets (A - B)** | The set of elements belonging to A but not to B, equivalent to A intersect B'. |
| **Fundamental Theory of Sets and Operations** | **Symmetric Difference (A delta B)** | Defined as (A - B) union (B - A) or (A union B) - (A intersect B). |
| **Cartesian Product and Binary Relations** | **De Morgan's Laws** | For any two subsets A and B of a universal set U, (A union B)' = A' intersect B' and (A intersect B)' = A' union B'. |
| **Functions, Mappings & Inverse Trigonometric Functions** | **For two finite sets A and B** | n(A union B) = n(A) + n(B) - n(A intersect B). |

| Core Sub-Domain | Key Concept / Provision | Verified Analytical Parameters & Exam Distinctions (UPSC Official Portal) |
|---|---|---|
| Fundamental Theory of Sets and Operations | For three finite sets A, B, and C | n(A union B union C) = n(A) + n(B) + n(C) - n(A intersect B) - n(B intersect C) - n(C intersect A) + n(A intersect B intersect C). |
| Cartesian Product and Binary Relations | Number of elements in exactly one of the three sets | n(A) + n(B) + n(C) - 2*n(A intersect B)* - 2n(B intersect C) - 2*n(C intersect A)* + 3n(A intersect B intersect C). |
| Functions, Mappings & Inverse Trigonometric Functions | Number of elements in exactly two of the three sets | n(A intersect B) + n(B intersect C) + n(C intersect A) - 3*n(A intersect B intersect C). |
| Fundamental Theory of Sets and Operations | Reflexive Relation | A relation R on set A is reflexive if (a, a) belongs to R for every element a in A. Total number of reflexive relations on A is 2^(n*(n - 1)). |
| Cartesian Product and Binary Relations | Symmetric Relation | A relation R is symmetric if whenever (a, b) belongs to R, then (b, a) also belongs to R. Total number of symmetric relations on A is 2^(n*(n + 1)/2). |

## Chapter *1* Multi-Statement Exam Verification Drills

**Chapter 1 Statement-Verification Drill #1** — Both Statements Verified Correct

**Statement-I (Representation of Sets):** Sets are represented either in Roster Form (Tabular Form), where elements are listed explicitly within braces such as A = {2, 3, 5, 7}, or in Set-Builder Form (Rule Method), where a characterizing property defines membership, such as A = {x : x is a prime number less than 10}.

**Statement-II (Cardinality and Subsets):** The cardinal number of a finite set A, denoted as n(A) or |A|, represents the total count of distinct elements in A. If every element of set A is also an element of set B, then A is a subset of B (A subset_of B). For a finite set with n elements, the total number of subsets is 2^n, the total number of proper subsets is (2^n - 1), and the total number of non-empty proper subsets is (2^n - 2).

✓ **Exam Verification Key: Both Statement-I and Statement-II are factually accurate as per UPSC Official Portal & IUC Academy.**

**Chapter 1 Statement-Verification Drill #2** — Both Statements Verified Correct

**Statement-I (Power Set):** The collection of all possible subsets of a set A is called its Power Set, denoted by P(A). If n(A) = m, then n(P(A)) = 2^m, and n(P(P(A))) = 2^(2^m).

**Statement-II (Union of Sets (A union B)):** The set of elements belonging to A, or B, or both.

✓ **Exam Verification Key: Both Statement-I and Statement-II are factually accurate as per UPSC Official Portal & IUC Academy.**

**Chapter 1 Statement-Verification Drill #3** — Both Statements Verified Correct

**Statement-I (Intersection of Sets (A intersect B)):** The set of elements common to both A and B. Two sets are disjoint if (A intersect B) = phi (empty set).

**Statement-II (Difference of Sets (A - B)):** The set of elements belonging to A but not to B, equivalent to A intersect B'.

✓ **Exam Verification Key: Both Statement-I and Statement-II are factually accurate as per UPSC Official Portal & IUC Academy.**

**Chapter 1 Statement-Verification Drill #4** — Both Statements Verified Correct

**Statement-I (Symmetric Difference (A delta B)):** Defined as (A - B) union (B - A) or (A union B) - (A intersect B).

**Statement-II (De Morgan's Laws):** For any two subsets A and B of a universal set U, (A union B)' = A' intersect B' and (A intersect B)' = A' union B'.

✓ **Exam Verification Key: Both Statement-I and Statement-II are factually accurate as per UPSC Official Portal & IUC Academy.**

**Chapter 1 Statement-Verification Drill #5** — Both Statements Verified Correct

**Statement-I (For two finite sets A and B):** n(A union B) = n(A) + n(B) - n(A intersect B).

**Statement-II (For three finite sets A, B, and C):** n(A union B union C) = n(A) + n(B) + n(C) - n(A intersect B) - n(B intersect C) - n(C intersect A) + n(A intersect B intersect C).

✓ **Exam Verification Key: Both Statement-I and Statement-II are factually accurate as per UPSC Official Portal & IUC Academy.**

**Chapter 1 Statement-Verification Drill #6** — Both Statements Verified Correct

**Statement-I (Number of elements in exactly one of the three sets):** n(A) + n(B) + n(C) - 2*n*(*A intersect B*) - 2n(B intersect C) - 2*n*(*C intersect A*) + 3n(A intersect B intersect C).

**Statement-II (Number of elements in exactly two of the three sets):** n(A intersect B) + n(B intersect C) + n(C intersect A) - 3*n(A intersect B intersect C).

✓ **Exam Verification Key: Both Statement-I and Statement-II are factually accurate as per UPSC Official Portal & IUC Academy.**

# Chapter *2:* Complex Numbers, Quadratic Equations & Sequences/Series

**Chapter Key Study Pillars — Chapter 2: Complex Numbers, Quadratic Equations & Sequences/Series**

| Key Study Pillar 1 | Key Study Pillar 2 | Key Study Pillar 3 |
|---|---|---|
| **Algebra and Geometry of Complex Numbers** | **Theory of Quadratic Equations and Nature of Roots** | **Arithmetic, Geometric, and Harmonic Progressions (AP, GP, HP)** |

## *2.1* Algebra and Geometry of Complex Numbers

A complex number z is expressed in standard Cartesian form as z = x + i*y, where x = Re(z) and y = Im(z) are real numbers, and i = sqrt(-1) is the imaginary unit satisfying i^2 = -1, i^3 = -i, and i^4 = 1. For any four consecutive integral powers of iota, i^n + i^(n+1) + i^(n+2) + i^(n+3) = 0 (**NCERT**).

• **Modulus, Conjugate, and Argument:**

• **Modulus:** |z| = sqrt(x^2 + y^2) represents the Euclidean distance of point (x, y) from the origin in the Argand plane. Key properties: |z1 *z2*| = |z1| |z2|, |z1 / z2| = |z1| / |z2|, and the Triangle Inequality ||z1| - |z2|| <= |z1 + z2| <= |z1| + |z2|.

• **Conjugate:** z_bar = x - i*y is the reflection of z across the real axis. Note that z* z_bar = |z|^2, Re(z) = (z + z_bar)/2, and Im(z) = (z - z_bar)/(2*i).

• **Principal Argument (Arg(z)):** The unique angle theta in (-pi, pi] that the vector OP makes with the positive real axis. Let alpha = arctan(|y/x|). In Quadrant I (x > 0, y > 0), Arg(z) = alpha; in Quadrant II (x < 0, y > 0), Arg(z) = pi - alpha; in Quadrant III (x < 0, y < 0), Arg(z) = -(pi - alpha) = alpha - pi; in Quadrant IV (x > 0, y < 0), Arg(z) = -alpha.

• **Polar Form, Euler's Form, and De Moivre's Theorem:**

• **Polar & Euler Representation:** z = r(*cos*(*theta*) + *i*sin(theta)) = r *e*^(*i*theta), where r = |z| and theta = Arg(z).

• **De Moivre's Theorem:** For any rational number n, (cos(theta) + i*sin*(*theta*))^*n* = *cos*(*n*theta) + i*sin*(*n*theta).

• **Cube Roots of Unity (1, omega, omega^2):**

• Solutions to z^3 = 1 are 1, omega = (-1 + i*sqrt*(3))/2 = *e*^(*i*2*pi*/3), *and omega*^2 = (-1 - *i*sqrt(3))/2 = e^(i4pi/3).

• Fundamental Identities: 1 + omega + omega^2 = 0 and omega^3 = 1. Furthermore, omega^2 = omega_bar = 1/omega. In the Argand plane, 1, omega, and omega^2 form the vertices of an equilateral triangle inscribed in the unit circle |z| = 1, with each side of length sqrt(3).

## *2.2* Theory of Quadratic Equations and Nature of Roots

A polynomial equation of degree two is written as a *x*^2 + *b*x + c = 0 (where a != 0). Its roots alpha and beta are given by the quadratic formula x = (-b +/- sqrt(D)) / (2*a*), *where D* = *b*^2 - 4a*c is the discriminant (**UPSC**).

• **Vieta's Formulas and Symmetric Functions of Roots:**

• Sum of roots: alpha + beta = -b/a.

• Product of roots: alpha * beta = c/a.

• Difference of roots: |alpha - beta| = sqrt((alpha + beta)^2 - 4*alpha*beta) = sqrt(D) / |a|.

• Sum of squares: alpha^2 + beta^2 = (alpha + beta)^2 - 2*alpha*beta = (b^2 - 2ac) / a^2.

• Sum of cubes: alpha^3 + beta^3 = (alpha + beta)^3 - 3*alpha*beta*(alpha + beta).

• **Condition for Common Roots:** Given a1*x*^2 + *b*1x + c1 = 0 and a2*x*^2 + *b*2x + c2 = 0:

• **Both Roots Common:** a1/a2 = b1/b2 = c1/c2.

• **Exactly One Root Common:** (c1*a*2 - *c*2a1)^2 = (b1*c*2 - *b*2c1)(*a*1b2 - a2*b1).

• **Location of Roots and Quadratic Expression Range:** For y = f(x) = a*x*^2 + *b*x + c, the graph is a parabola with vertex at (-b/(2*a*), -*D*/(4a)). If a > 0, the minimum value is -D/(4*a*) *at x* = -*b*/(2a); if a < 0, the maximum value is -D/(4*a*) *at x* = -*b*/(2a).

## *2.3* Arithmetic, Geometric, and Harmonic Progressions (AP, GP, HP)

Sequences following fixed mathematical patterns are heavily tested in **UPSC NDA** and **IUC Academy** mock assessments.

• **Arithmetic Progression (AP):** nth term T_n = a + (n - 1)*d*; *sum of n terms S_n* = (*n*/2)[2*a* + (*n* - 1)d] = (n/2)*(a + l). Arithmetic Mean (AM) of a and b is (a + b)/2.

• **Geometric Progression (GP):** nth term T_n = a *r*^(*n* - 1); *sum of n terms S_n* = *a*(1 - r^n)/(1 - r) for r != 1; sum of infinite GP S_infinity = a / (1 - r) when |r| < 1. Geometric Mean (GM) of positive numbers a and b is sqrt(a*b).

• **Harmonic Progression (HP):** Reciprocals of terms form an AP. Harmonic Mean (HM) of a and b is 2*a*b / (a + b).

• **AM-GM-HM Inequality:** For any positive real numbers, AM >= GM >= HM, with equality holding if and only if all numbers are equal. For two positive numbers a and b, GM^2 = AM *HM* (*i.e.*, *G*^2 = *A* H).

- **Special Power Series Summations:**
- Sum of first n natural numbers: sum(k) = n*(n + 1)/2.
- Sum of squares of first n natural numbers: sum(k^2) = n(*n* + 1)(2*n + 1)/6.
- Sum of cubes of first n natural numbers: sum(k^3) = [n*(n + 1)/2]^2.

**Step-by-Step Conceptual Process Flowchart**

**STEP 1**
**Convert Complex Number to Standard Form x + iy**

**STEP 2**
**Compute Modulus r and Principal Argument theta**

**STEP 3**
**Apply De Moivre Theorem or Cube Root of Unity Identities**

**STEP 4**
**Verify Locus or Root Conditions via Discriminant & AM-GM**

| Locus Equation / Algebraic Condition | Geometric / Mathematical Interpretation | Key Parameter / Formula | Official Source | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| ** | z - z0 | = r** | Circle with center z0 and radius r | Area = pi * r^2 | NCERT Class XI | | | | | |
| ** | z - z1 | = | z - z2 | ** | Perpendicular bisector of segment joining z1 and z2 | Straight line locus | UPSC NDA Portal | | | |
| ** | z - z1 | + | z - z2 | = k (k > | z1 - z2 | )** | Ellipse with foci at z1 and z2, major axis 2a = k | Eccentricity e = | z1 - z2 | / k |
| ** | z - z1 | + | z - z2 | = | z1 - z2 | ** | Line segment joining z1 and z2 | Degenerate ellipse | IUC Academy | |
| ** | | z - z1 | - | z - z2 | | = k (k < | z1 - z2 | )** | Hyperbola with foci at z1 and z2 | Transv axis = |

| Locus Equation / Algebraic Condition | Geometric / Mathematical Interpretation | Key Parameter / Formula | Official Source | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Arg((z - z1)/(z - z2)) = +/- pi/2** | Circle with segment joining z1 and z2 as diameter | | z - z1 | ^2 + | z - z2 | ^2 = | z1 - z2 | ^2 | NCERT Class XI |

**Did You Know?:** If the roots of a quadratic equation with rational coefficients are irrational, they always occur in conjugate pairs p + sqrt(q) and p - sqrt(q); similarly, if a quadratic equation has real coefficients and a negative discriminant (D < 0), its complex roots always occur in conjugate pairs alpha + i*beta and alpha - i*beta (NCERT).

**Exam Trap Alert:** The algebraic identity sqrt(a) *sqrt(b) = sqrt(ab) fails when both a and b are negative real numbers! Specifically, sqrt(-4) *sqrt(-9) = (2i)(3i) = 6i^2 = -6, *whereas carelessly writing sqrt((-4)(-9)) = sqrt(36) = +6* yields the wrong sign (UPSC).

★ **Memory Trick:** Remember the power-reduction shortcut for ((1 + i)/(1 - i)) and ((1 - i)/(1 + i)): multiplying numerator and denominator by the conjugate immediately simplifies **(1 + i)/(1 - i) = i** and **(1 - i)/(1 + i) = -i**, turning high-power expressions like ((1 + i)/(1 - i))^100 into a 2-second calculation i^100 = 1 (IUC Academy).

## Chapter 2 Comprehensive Analytical & Statutory Revision Matrix

| Core Sub-Domain | Key Concept / Provision | Verified Analytical Parameters & Exam Distinctions (UPSC Official Portal) |
|---|---|---|
| **Algebra and Geometry of Complex Numbers** | **Modulus** | \|z\| = sqrt(x^2 + y^2) represents the Euclidean distance of point (x, y) from the origin in the Argand plane. Key properties: \|z1 z2\| = \|z1\| \|z2\|, \|z1 / z2\| = \|z1\| / \|z2\|, and the Triangle Inequality \|\|z1\| - \|z2\|\| <= \|z1 + z2\| <= \|z1\| + \|z2\|. |
| **Theory of Quadratic Equations and Nature of Roots** | **Conjugate** | z_bar = x - iy *is the reflection of z across the real axis. Note that z* z_bar = \|z\| ^2, Re(z) = (z + z_bar)/2, and Im(z) = (z - z_bar)/(2*i). |
| **Arithmetic, Geometric, and Harmonic Progressions (AP, GP, HP)** | **Principal Argument (Arg(z))** | The unique angle theta in (-pi, pi] that the vector OP makes with the positive real axis. Let alpha = arctan( \|y/x\| ). In Quadrant I (x > 0, y > 0), Arg(z) = alpha; in Quadrant II (x < 0, y > 0), Arg(z) = pi - alpha; in Quadrant III (x < 0, y < 0), Arg(z) = -(pi - alpha) = alpha - pi; in Quadrant IV (x > 0, y < 0), Arg(z) = -alpha. |
| **Algebra and Geometry of Complex Numbers** | **Polar & Euler Representation** | z = r(*cos(theta)* + *i*sin(theta)) = r *e^(i*theta), where r = \|z\| and theta = Arg(z). |
| **Theory of Quadratic Equations and Nature of Roots** | **De Moivre's Theorem** | For any rational number n, (cos(theta) + i*sin(theta))^n = *cos(n*theta) + i*sin(n*theta). |

| Core Sub-Domain | Key Concept / Provision | Verified Analytical Parameters & Exam Distinctions (UPSC Official Portal) |
|---|---|---|
| Arithmetic, Geometric, and Harmonic Progressions (AP, GP, HP) | Fundamental Identities | 1 + omega + omega^2 = 0 and omega^3 = 1. Furthermore, omega^2 = omega_bar = 1/omega. In the Argand plane, 1, omega, and omega^2 form the vertices of an equilateral triangle inscribed in the unit circle \|z\| = 1, with each side of length sqrt(3). |
| Algebra and Geometry of Complex Numbers | Sum of roots | alpha + beta = -b/a. |
| Theory of Quadratic Equations and Nature of Roots | Difference of roots | \|alpha - beta\| = sqrt((alpha + beta)^2 - 4*alpha*beta) = sqrt(D) / \|a\|. |
| Arithmetic, Geometric, and Harmonic Progressions (AP, GP, HP) | Sum of squares | alpha^2 + beta^2 = (alpha + beta)^2 - 2*alpha*beta = (b^2 - 2ac) / a^2. |
| Algebra and Geometry of Complex Numbers | Sum of cubes | alpha^3 + beta^3 = (alpha + beta)^3 - 3*alpha*beta*(alpha + beta). |
| Theory of Quadratic Equations and Nature of Roots | Condition for Common Roots | Given a1*x^2 + *b1x + c1 = 0 and a2*x^2 + *b2x + c2 = 0: |
| Arithmetic, Geometric, and Harmonic Progressions (AP, GP, HP) | Both Roots Common | a1/a2 = b1/b2 = c1/c2. |
| Algebra and Geometry of Complex Numbers | Exactly One Root Common | (c1*a2 - *c2a1)^2 = (b1*c2 - *b2c1)(*a1b2 - a2*b1). |
| Theory of Quadratic Equations and Nature of Roots | Location of Roots and Quadratic Expression Range | For y = f(x) = a*x^2 + *bx + c, the graph is a parabola with vertex at (-b/(2*a*), -*D*/(4a)). If a > 0, the minimum value is -D/(4*a*) *at x* = -*b*/(2a); if a < 0, the maximum value is -D/(4*a*) *at x* = -*b*/(2a). |

## Chapter 2 Multi-Statement Exam Verification Drills

**Chapter 2 Statement-Verification Drill #1** — Both Statements Verified Correct

**Statement-I (Modulus):** |z| = sqrt(x^2 + y^2) represents the Euclidean distance of point (x, y) from the origin in the Argand plane. Key properties: |z1 z2| = |z1| |z2|, |z1 / z2| = |z1| / |z2|, and the Triangle Inequality ||z1| - |z2|| <= |z1 + z2| <= |z1| + |z2|.

**Statement-II (Conjugate):** z_bar = x - iy *is the reflection of z across the real axis. Note that* z z_bar = |z|^2, Re(z) = (z + z_bar)/2, and Im(z) = (z - z_bar)/(2*i).

✓ **Exam Verification Key: Both Statement-I and Statement-II are factually accurate as per UPSC Official Portal & IUC Academy.**

**Chapter 2 Statement-Verification Drill #2** — Both Statements Verified Correct

**Statement-I (Principal Argument (Arg(z))):** The unique angle theta in (-pi, pi] that the vector OP makes with the positive real axis. Let alpha = arctan(|y/x|). In Quadrant I (x > 0, y > 0), Arg(z) = alpha; in Quadrant II (x < 0, y > 0), Arg(z) = pi - alpha; in Quadrant III (x < 0, y < 0), Arg(z) = -(pi - alpha) = alpha - pi; in Quadrant IV (x > 0, y < 0), Arg(z) = -alpha.

**Statement-II (Polar & Euler Representation):** z = r(*cos*(*theta*) + *i*sin(theta)) = r *e*^(*i*theta), where r = |z| and theta = Arg(z).

✓ **Exam Verification Key: Both Statement-I and Statement-II are factually accurate as per UPSC Official Portal & IUC Academy.**

**Chapter 2 Statement-Verification Drill #3** — Both Statements Verified Correct

**Statement-I (De Moivre's Theorem):** For any rational number n, (cos(theta) + i*sin*(*theta*))^*n* = *cos*(*n*theta) + i*sin*(*n*theta).

**Statement-II (Fundamental Identities):** 1 + omega + omega^2 = 0 and omega^3 = 1. Furthermore, omega^2 = omega_bar = 1/omega. In the Argand plane, 1, omega, and omega^2 form the vertices of an equilateral triangle inscribed in the unit circle |z| = 1, with each side of length sqrt(3).

✓ **Exam Verification Key: Both Statement-I and Statement-II are factually accurate as per UPSC Official Portal & IUC Academy.**

**Chapter 2 Statement-Verification Drill #4** — Both Statements Verified Correct

**Statement-I (Sum of roots):** alpha + beta = -b/a.

**Statement-II (Difference of roots):** |alpha - beta| = sqrt((alpha + beta)^2 - 4*alpha*beta) = sqrt(D) / |a|.

✓ **Exam Verification Key: Both Statement-I and Statement-II are factually accurate as per UPSC Official Portal & IUC Academy.**

**Chapter 2 Statement-Verification Drill #5** — Both Statements Verified Correct

**Statement-I (Sum of squares):** alpha^2 + beta^2 = (alpha + beta)^2 - 2*alpha*beta = (b^2 - 2*ac*) / a^2.

**Statement-II (Sum of cubes):** alpha^3 + beta^3 = (alpha + beta)^3 - 3*alpha*beta*(alpha + beta).

✓ **Exam Verification Key: Both Statement-I and Statement-II are factually accurate as per UPSC Official Portal & IUC Academy.**

**Chapter 2 Statement-Verification Drill #6** — Both Statements Verified Correct

**Statement-I (Condition for Common Roots):** Given a1*x*^2 + *b*1x + c1 = 0 and a2*x*^2 + *b*2x + c2 = 0:

**Statement-II (Both Roots Common):** a1/a2 = b1/b2 = c1/c2.

✓ **Exam Verification Key: Both Statement-I and Statement-II are factually accurate as per UPSC Official Portal & IUC Academy.**

# Chapter 3: Matrices, Determinants & Systems of Linear Equations

**Chapter Key Study Pillars — Chapter 3: Matrices, Determinants & Systems of Linear Equations**

| Key Study Pillar 1 | Key Study Pillar 2 | Key Study Pillar 3 |
|---|---|---|
| Matrix Algebra, Special Matrices & Transpose Properties | Determinants, Minors, Cofactors & Adjoint Properties | Systems of Linear Equations & Cramer's Rule |

## 3.1 Matrix Algebra, Special Matrices & Transpose Properties

A matrix A = [a_ij] of order m x n is a rectangular array of m horizontal rows and n vertical columns containing m*n elements. Matrices are indispensable for solving multi-variable linear systems in engineering and defence examinations (**UPSC**, **NCERT**).

- **Classification of Square Matrices (Order n x n):**
- **Diagonal Matrix:** All non-diagonal elements are zero (a_ij = 0 for i != j).
- **Scalar Matrix:** A diagonal matrix where all diagonal elements are equal (a_ii = k).
- **Identity (Unit) Matrix (I_n):** A scalar matrix with all diagonal elements equal to 1. Satisfies A *I* = *I* A = A and I^k = I.
- **Trace of a Matrix (tr(A)):** Sum of the principal diagonal elements, tr(A) = sum_{i=1 to n} a_ii. Note that tr(A + B) = tr(A) + tr(B) and tr(A*B*) = *tr*(*B*A).
- **Symmetric Matrix:** A^T = A (i.e., a_ij = a_ji). Total independent elements in an n x n symmetric matrix is n*(n + 1)/2.
- **Skew-Symmetric Matrix:** A^T = -A (i.e., a_ij = -a_ji). All principal diagonal elements of a skew-symmetric matrix MUST be zero (a_ii = 0). Total independent elements is n*(n - 1)/2. Every square matrix A can be uniquely decomposed as the sum of a symmetric matrix P = (A + A^T)/2 and a skew-symmetric matrix Q = (A - A^T)/2.
- **Orthogonal Matrix:** A *A^T* = *A^T* A = I, which means A^(-1) = A^T and det(A) = +/- 1.
- **Idempotent, Involutory, and Nilpotent Matrices:** Idempotent satisfies A^2 = A (det(A) = 0 or 1); Involutory satisfies A^2 = I (so A^(-1) = A); Nilpotent satisfies A^k = O for some positive integer k (called the index of nilpotency, and det(A) = 0 always).

## 3.2 Determinants, Minors, Cofactors & Adjoint Properties

Every square matrix A of order n x n is associated with a scalar value called its determinant, denoted det(A) or |A| (**NCERT**).

- **Key Determinant Properties for Order n:**
- |A^T| = |A|, and |A *B*| = |*A*| |B|.
- If each element of a row (or column) is multiplied by a scalar k, the determinant is multiplied by k. Hence, if an entire n x n matrix A is multiplied by scalar k, then ***|k A| = k^n |A|***.
- Interchanging any two rows (or columns) changes the sign of the determinant. If two rows or columns are identical or proportional, |A| = 0.

• The determinant of a skew-symmetric matrix of **odd order** is always **0**, while the determinant of a skew-symmetric matrix of **even order** is always a **non-zero perfect square**.

• **Adjoint and Inverse of a Square Matrix:**

• The adjoint of matrix A, denoted adj(A), is the transpose of its cofactor matrix: adj(A) = [C_ij]^T.

• Fundamental Identity: **A *adj(A)* = *adj(A)* A = |A| * I_n**.

• A square matrix A is invertible (non-singular) if and only if |A| != 0, and its inverse is given by **A^(-1) = adj(A) / |A|**.

• High-Yield Adjoint Formulas (for order n x n with |A| != 0):

• **|adj(A)| = |A|^(n - 1)**

• **adj(adj(A)) = |A|^(n - 2) * A**

• **|adj(adj(A))| = |A|^((n - 1)^2)**

• **adj(k *A*) = *k*^(*n* - 1) adj(A)**

• **adj(A *B*) = *adj*(*B*) adj(A)** (Reversal Law)

## *3.3* Systems of Linear Equations & Cramer's Rule

Consider a system of three linear equations in three variables: a1*x* + *b*1y + c1*z* = *d*1, *a*2x + b2*y* + *c*2z = d2, a3*x* + *b*3y + c3*z* = *d*3, *represented in matrix form as A* X = B, or via Cramer's Rule determinants Delta = |A|, Delta_x, Delta_y, Delta_z (IUC Academy).

• **Non-Homogeneous System (B != O):**

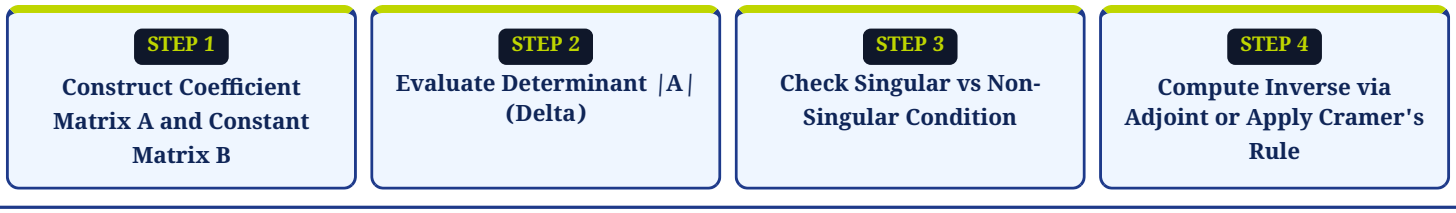
• **Case 1:** If Delta != 0 (i.e., |A| != 0), the system is **Consistent** and has a **Unique Solution** given by X = A^(-1) * B, or x = Delta_x / Delta, y = Delta_y / Delta, z = Delta_z / Delta.

• **Case 2:** If Delta = 0 and at least one of Delta_x, Delta_y, Delta_z is non-zero (i.e., adj(A) * B != O), the system is **Inconsistent** (No Solution).

• **Case 3:** If Delta = 0 and Delta_x = Delta_y = Delta_z = 0 (and adj(A) * B = O), the system may have either **Infinitely Many Solutions** (consistent) or no solution (if planes are parallel).

• **Homogeneous System (A * X = O, where d1 = d2 = d3 = 0):** Always consistent because x = y = z = 0 (the trivial solution) always satisfies it. If |A| != 0, it has ONLY the trivial solution (0, 0, 0). If |A| = 0, it possesses **Infinitely Many Non-Trivial Solutions**.

**Step-by-Step Conceptual Process Flowchart**



| Matrix / Determinant Operation | Formula for Order n x n Square Matrix A | Result for Order n = 3 and | A | = 5 | Official Source | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **Scalar Multiplication | k * A | ** | k^n * | A | | For k = 2: 2^3 * 5 = 40 | NCERT Class XII | | | |
| **Determinant of Adjoint | adj(A) | ** | | A | ^(n - 1) | 5^(3 - 1) = 5^2 = 25 | UPSC NDA Portal | | | |
| **Determinant of Inverse | A^(-1) | ** | 1 / | A | = | A | ^(-1) | 1 / 5 = 0.2 | NCERT Class XII | |
| **Determinant of Double Adjoint | adj(adj(A)) | ** | | A | ^((n - 1)^2) | 5^((3 - 1)^2) = 5^4 = 625 | UPSC NDA Portal | | | |
| **Determinant of Product | A * adj(A) | ** | | | A | * I_n | = | A | ^n | 5^3 = 125 | IUC Academy |
| **Adjoint of Scalar Multiple | adj(k * A) | ** | k^(n(*n* - 1)) | A | ^(n - 1) | For k = 2: 2^6 * 5^2 = 1600 | IUC Academy | | | |

**Did You Know?:** Matrix multiplication is NOT commutative in general (A*B* != *B*A), which implies that the algebraic expansion (A + B)^2 = A^2 + 2*A*B + B^2 holds if and only if A and B commute (i.e., A*B* = *B*A); otherwise, (A + B)^2 = A^2 + A*B* + *B*A + B^2 (NCERT).

**Exam Trap Alert:** Do not confuse scalar multiplication of a matrix with scalar multiplication of a determinant! In a matrix, multiplying by k multiplies EVERY element of all n rows by k, so |k*A*| = *k*^*n* |A|. In a determinant, multiplying by k multiplies only a SINGLE row or column by k (UPSC).

★ **Memory Trick:** To instantly find the inverse of a 2 x 2 matrix A = [[a, b], [c, d]], use **"Swap the Main, Sign-Flip the Off"**: swap the main diagonal elements (a and d), negate the off-diagonal elements (-b and -c), and divide by (a*d* - *b*c) to get A^(-1) = (1/(a*d* - *b*c)) * [[d, -b], [-c, a]] (IUC Academy).

## Chapter 3 Comprehensive Analytical & Statutory Revision Matrix

| Core Sub-Domain | Key Concept / Provision | Verified Analytical Parameters & Exam Distinctions (UPSC Official Portal) |
|---|---|---|
| **Matrix Algebra, Special Matrices & Transpose Properties** | **Diagonal Matrix** | All non-diagonal elements are zero (a_ij = 0 for i != j). |
| **Determinants, Minors, Cofactors & Adjoint Properties** | **Scalar Matrix** | A diagonal matrix where all diagonal elements are equal (a_ii = k). |
| **Systems of Linear Equations & Cramer's Rule** | **Identity (Unit) Matrix (I_n)** | A scalar matrix with all diagonal elements equal to 1. Satisfies A *I* = *I* A = A and I^k = I. |
| **Matrix Algebra, Special Matrices & Transpose Properties** | **Trace of a Matrix (tr(A))** | Sum of the principal diagonal elements, tr(A) = sum_{i=1 to n} a_ii. Note that tr(A + B) = tr(A) + tr(B) and tr(A*B*) = *tr*(*B*A). |
| **Determinants, Minors, Cofactors & Adjoint Properties** | **Symmetric Matrix** | A^T = A (i.e., a_ij = a_ji). Total independent elements in an n x n symmetric matrix is n*(n + 1)/2. |
| **Systems of Linear Equations & Cramer's Rule** | **Skew-Symmetric Matrix** | A^T = -A (i.e., a_ij = -a_ji). All principal diagonal elements of a skew-symmetric matrix MUST be zero (a_ii = 0). Total independent elements is n*(n - 1)/2. Every square matrix A can be uniquely decomposed as the sum of a symmetric matrix P = (A + A^T)/2 and a skew-symmetric matrix Q = (A - A^T)/2. |
| **Matrix Algebra, Special Matrices & Transpose Properties** | **Orthogonal Matrix** | A *A^T* = *A^T* A = I, which means A^(-1) = A^T and det(A) = +/- 1. |
| **Determinants, Minors, Cofactors & Adjoint Properties** | **Idempotent, Involutory, and Nilpotent Matrices** | Idempotent satisfies A^2 = A (det(A) = 0 or 1); Involutory satisfies A^2 = I (so A^(-1) = A); Nilpotent satisfies A^k = O for some positive integer k (called the index of nilpotency, and det(A) = 0 always). |
| **Systems of Linear Equations & Cramer's Rule** | **Fundamental Identity** | **A *adj(A)* = *adj(A)* A = \|A\| * I_n**. |
| **Matrix Algebra, Special Matrices & Transpose Properties** | **Case 1** | If Delta != 0 (i.e., \|A\| != 0), the system is **Consistent** and has a **Unique Solution** given by X = A^(-1) * B, or x = Delta_x / Delta, y = Delta_y / Delta, z = Delta_z / Delta. |
| **Determinants, Minors, Cofactors & Adjoint Properties** | **Case 2** | If Delta = 0 and at least one of Delta_x, Delta_y, Delta_z is non-zero (i.e., adj(A) * B != O), the system is **Inconsistent** (No Solution). |

| Core Sub-Domain | Key Concept / Provision | Verified Analytical Parameters & Exam Distinctions (UPSC Official Portal) |
|---|---|---|
| **Systems of Linear Equations & Cramer's Rule** | **Case 3** | If Delta = 0 and Delta_x = Delta_y = Delta_z = 0 (and adj(A) * B = O), the system may have either **Infinitely Many Solutions** (consistent) or no solution (if planes are parallel). |
| **Matrix Algebra, Special Matrices & Transpose Properties** | **Homogeneous System (A * X = O, where d1 = d2 = d3 = 0)** | **Always consistent because x = y = z = 0 (the trivial solution) always satisfies it. If \|A\| != 0, it has ONLY the trivial solution (0, 0, 0). If \|A\| = 0, it possesses** Infinitely Many Non-Trivial Solutions**. |

## Chapter *3* Multi-Statement Exam Verification Drills

**Chapter 3 Statement-Verification Drill #1** — Both Statements Verified Correct

**Statement-I (Diagonal Matrix):** All non-diagonal elements are zero (a_ij = 0 for i != j).
**Statement-II (Scalar Matrix):** A diagonal matrix where all diagonal elements are equal (a_ii = k).

✓ **Exam Verification Key: Both Statement-I and Statement-II are factually accurate as per UPSC Official Portal & IUC Academy.**

**Chapter 3 Statement-Verification Drill #2** — Both Statements Verified Correct

**Statement-I (Identity (Unit) Matrix (I_n)):** A scalar matrix with all diagonal elements equal to 1. Satisfies A *I* = *I* A = A and I^k = I.
**Statement-II (Trace of a Matrix (tr(A))):** Sum of the principal diagonal elements, tr(A) = sum_{i=1 to n} a_ii. Note that tr(A + B) = tr(A) + tr(B) and tr(A*B*) = *tr*(*B*A).

✓ **Exam Verification Key: Both Statement-I and Statement-II are factually accurate as per UPSC Official Portal & IUC Academy.**

**Chapter 3 Statement-Verification Drill #3** — Both Statements Verified Correct

**Statement-I (Symmetric Matrix):** A^T = A (i.e., a_ij = a_ji). Total independent elements in an n x n symmetric matrix is n*(n + 1)/2.
**Statement-II (Skew-Symmetric Matrix):** A^T = -A (i.e., a_ij = -a_ji). All principal diagonal elements of a skew-symmetric matrix MUST be zero (a_ii = 0). Total independent elements is n*(n - 1)/2. Every square matrix A can be uniquely decomposed as the sum of a symmetric matrix P = (A + A^T)/2 and a skew-symmetric matrix Q = (A - A^T)/2.

✓ **Exam Verification Key: Both Statement-I and Statement-II are factually accurate as per UPSC Official Portal & IUC Academy.**

**Chapter 3 Statement-Verification Drill #4** — Both Statements Verified Correct

**Statement-I (Orthogonal Matrix):** A *A^T* = *A^T* A = I, which means A^(-1) = A^T and det(A) = +/- 1.
**Statement-II (Idempotent, Involutory, and Nilpotent Matrices):** Idempotent satisfies A^2 = A (det(A) = 0 or 1); Involutory satisfies A^2 = I (so A^(-1) = A); Nilpotent satisfies A^k = O for some positive integer k (called the index of nilpotency, and det(A) = 0 always).

✓ **Exam Verification Key: Both Statement-I and Statement-II are factually accurate as per UPSC Official Portal & IUC Academy.**

**Chapter 3 Statement-Verification Drill #5** — Both Statements Verified Correct

**Statement-I (Fundamental Identity): A *adj(A)* = *adj(A)* A = \|A\| * I_n.**
**Statement-II (Case 1):** If Delta != 0 (i.e., \|A\| != 0), the system is **Consistent** and has a **Unique Solution** given by X = A^(-1) * B, or x = Delta_x / Delta, y = Delta_y / Delta, z = Delta_z / Delta.

✓ **Exam Verification Key: Both Statement-I and Statement-II are factually accurate as per UPSC Official Portal & IUC Academy.**

**Chapter 3 Statement-Verification Drill #6** Both Statements Verified Correct

**Statement-I (Case 2):** If Delta = 0 and at least one of Delta_x, Delta_y, Delta_z is non-zero (i.e., adj(A) * B != O), the system is **Inconsistent** (No Solution).

**Statement-II (Case 3):** If Delta = 0 and Delta_x = Delta_y = Delta_z = 0 (and adj(A) * B = O), the system may have either **Infinitely Many Solutions** (consistent) or no solution (if planes are parallel).

✓ **Exam Verification Key: Both Statement-I and Statement-II are factually accurate as per UPSC Official Portal & IUC Academy.**

# Chapter 4: Permutations, Combinations & Binomial Theorem

**Chapter Key Study Pillars — Chapter 4: Permutations, Combinations & Binomial Theorem**

**Key Study Pillar 1**
**Fundamental Principle of Counting, Permutations & Combinations**

**Key Study Pillar 2**
**Binomial Theorem for Positive Integral Index**

## 4.1 Fundamental Principle of Counting, Permutations & Combinations

Combinatorics governs the systematic counting of arrangements (permutations, where order matters) and selections (combinations, where order does not matter), forming a core pillar of the UPSC NDA and NCERT syllabi.

• **Permutations (Arrangement of r objects out of n distinct objects):**

• Without repetition: **nPr = n! / (n - r)!**.

• With repetition allowed: **n^r**.

• Permutations of n objects where p are alike of one kind, q are alike of a second kind, and r are alike of a third kind: **n! / (p! *q*! r!)**.

• **Circular Permutations:** The number of ways to arrange n distinct objects around a fixed circle is **(n - 1)!**. If clockwise and anticlockwise arrangements are indistinguishable (such as beads on a necklace or flowers in a garland), the number of arrangements is **(n - 1)! / 2** (for n >= 3).

• **Combinations (Selection of r objects out of n distinct objects):**

• Formula: **nCr = n! / [r! * (n - r)!] = nPr / r!**.

• Key Identities:

• Symmetry Property: **nCr = nC(n - r)**. Thus, if nCx = nCy, then either **x = y** or **x + y = n**.

• **Pascal's Identity: nCr + nC(r - 1) = (n + 1)Cr**.

• Ratio of consecutive combinations: **nCr / nC(r - 1) = (n - r + 1) / r**.

• Total ways of selecting one or more items from n distinct objects: **2^n - 1 = nC1 + nC2 + ... + nCn**.

• **Geometric & Special Counting Applications:**

• Given n points in a plane where p points are collinear:

• Number of straight lines = **nC2 - pC2 + 1**.

• Number of triangles = **nC3 - pC3**.

• Number of diagonals in a convex polygon of n sides = **nC2 - n = n*(n - 3)/2**.

• **Derangements (D_n):** The number of ways to place n distinct letters into n addressed envelopes such that NO letter goes into its correct envelope is **D_n = n! * [1 - 1/1! + 1/2! - 1/3! + ... + (-1)^n / n!]**. Key values: D_1 = 0, D_2 = 1, D_3 = 2, D_4 = 9, D_5 = 44.

## *4.2* Binomial Theorem for Positive Integral Index

For any positive integer n, the Binomial Expansion of (x + y)^n is given by:

**(x + y)^n = sum_{r=0 to n} [nCr *x*^(*n* - *r*) y^r] = nC0*x*^*n* + *n*C1x^(n-1)*y* + ... + *n*Cny^n** (NCERT).

• **Total Number of Terms:** In the expansion of (x + y)^n, there are **(n + 1)** terms. More generally, in the multinomial expansion of (x1 + x2 + ... + xk)^n, the total number of distinct terms is **(n + k - 1)C(k - 1)**.

• **General Term (r + 1)th Term: T_(r+1) = nCr *x*^(*n* - *r*) y^r**.

• **Independent Term (Constant Term) Shortcuts:** In the expansion of *(ax^p + b / x^q)^n, the value of r for the term containing x^k is given directly by* ***r* = (*n*p - k) / (p + q)**. For the **term independent of x** (constant term), put k = 0, so **r = (n*p) / (p + q)**.

• **Middle Term(s) in (x + y)^n:**

• If n is **even**, there is a single middle term: the **(n/2 + 1)th term**, with binomial coefficient nC(n/2).

• If n is **odd**, there are two middle terms: the **((n + 1)/2)th** and **((n + 3)/2)th terms**, with equal binomial coefficients nC((n - 1)/2) = nC((n + 1)/2).

• **Properties of Binomial Coefficients:**

• Sum of all binomial coefficients: **nC0 + nC1 + nC2 + ... + nCn = 2^n** (put x = 1, y = 1).

• Sum of even-indexed equals sum of odd-indexed coefficients: **nC0 + nC2 + nC4 + ... = nC1 + nC3 + nC5 + ... = 2^(n - 1)**.

• Weighted sum: **1*n*C1 + 2nC2 + 3*n*C3 + ... + *n*nCn = n * 2^(n - 1)**.

• Sum of squares: **(nC0)^2 + (nC1)^2 + ... + (nCn)^2 = (2n)Cn = (2n)! / (n!)^2**.

**Step-by-Step Conceptual Process Flowchart**

**STEP 1** Identify whether Order Matters (Permutation vs Combination)

**STEP 2** Apply Constraints (Gap Method, String Method, or Collinearity)

**STEP 3** Write General Term T_(r+1) for Binomial Expansion

**STEP 4** Equate Power of x to Target Exponent to Solve for r

| Combinatorial / Binomial Scenario | Mathematical Formula / Expression | Example Calculation | Official Reference |
|---|---|---|---|
| **Circular Arrangement (Necklace/ Garland)** | (n - 1)! / 2 | For n = 6 beads: 5! / 2 = 60 ways | NCERT Class XI |
| **Diagonals in an n-Sided Polygon** | n*(n - 3) / 2 | Octagon (n = 8): 8*5 / 2 = 20 diagonals | UPSC NDA Portal |
| **Derangement of n Items (D_n)** | n! * sum_{k=0 to n} [(-1)^k / k!] | For n = 4 letters: D_4 = 9 ways | IUC Academy |

| Combinatorial / Binomial Scenario | Mathematical Formula / Expression | Example Calculation | Official Reference |
|---|---|---|---|
| **Multinomial Terms in (x1 + x2 + ... + xk)^n** | (n + k - 1)C(k - 1) | In (a + b + c)^10 (n=10, k=3): 12C2 = 66 terms | UPSC NDA Portal |
| **Constant Term Index r in (a*x^p + b/x^q)^n** | r = (n * p) / (p + q) | In (x^2 + 1/x)^9: r = (9*2)/(2+1) = 6 (7th term) | IUC Academy |

**Did You Know?:** The sum of all coefficients in ANY polynomial or multinomial expansion f(x, y, z) is found simply by substituting x = 1, y = 1, z = 1 into the original expression! For example, the sum of all coefficients in the expansion of (3*x* - 2y + z)^8 is (3(1) - 2(1) + 1)^8 = 2^8 = 256 (IUC Academy).

**Exam Trap Alert:** Do not confuse the **"Sum of Binomial Coefficients"** with the **"Sum of Coefficients of the Expansion"**! In (3*x* - 5y)^n, the sum of binomial coefficients (nC0 + nC1 + ... + nCn) is always 2^n, whereas the sum of all coefficients in the expanded polynomial is obtained by setting x = 1, y = 1, which gives (3 - 5)^n = (-2)^n (UPSC).

★ **Memory Trick:** Use **"Tie Together vs. Slot Between"** for arrangement constraints: when specific objects MUST stay together, tie them into 1 super-object (**String Method**); when specific objects must NEVER sit together, arrange the remaining objects first and place the restricted objects into the gaps between them (**Gap Method**) (NCERT).

## Chapter *4* Comprehensive Analytical & Statutory Revision Matrix

| Core Sub-Domain | Key Concept / Provision | Verified Analytical Parameters & Exam Distinctions (UPSC Official Portal) |
|---|---|---|
| **Fundamental Principle of Counting, Permutations & Combinations** | **Without repetition** | **nPr = n! / (n - r)!**. |
| **Binomial Theorem for Positive Integral Index** | **Circular Permutations** | The number of ways to arrange n distinct objects around a fixed circle is **(n - 1)!**. If clockwise and anticlockwise arrangements are indistinguishable (such as beads on a necklace or flowers in a garland), the number of arrangements is **(n - 1)! / 2** (for n >= 3). |
| **Fundamental Principle of Counting, Permutations & Combinations** | **Formula** | **nCr = n! / [r! * (n - r)!] = nPr / r!**. |
| **Binomial Theorem for Positive Integral Index** | **Symmetry Property** | **nCr = nC(n - r)**. Thus, if nCx = nCy, then either **x = y** or **x + y = n**. |

| Core Sub-Domain | Key Concept / Provision | Verified Analytical Parameters & Exam Distinctions (UPSC Official Portal) |
|---|---|---|
| **Fundamental Principle of Counting, Permutations & Combinations** | **Pascal's Identity** | **nCr + nC(r - 1) = (n + 1)Cr**. |
| **Binomial Theorem for Positive Integral Index** | **Ratio of consecutive combinations** | **nCr / nC(r - 1) = (n - r + 1) / r**. |
| **Fundamental Principle of Counting, Permutations & Combinations** | **Total ways of selecting one or more items from n distinct objects** | **2^n - 1 = nC1 + nC2 + ... + nCn**. |
| **Binomial Theorem for Positive Integral Index** | **Derangements (D_n)** | The number of ways to place n distinct letters into n addressed envelopes such that NO letter goes into its correct envelope is **D_n = n! * [1 - 1/1! + 1/2! - 1/3! + ... + (-1)^n / n!]**. Key values: D_1 = 0, D_2 = 1, D_3 = 2, D_4 = 9, D_5 = 44. |
| **Fundamental Principle of Counting, Permutations & Combinations** | **Total Number of Terms** | In the expansion of (x + y)^n, there are **(n + 1)** terms. More generally, in the multinomial expansion of (x1 + x2 + ... + xk)^n, the total number of distinct terms is **(n + k - 1)C(k - 1)**. |
| **Binomial Theorem for Positive Integral Index** | **General Term (r + 1)th Term** | **T_(r+1) = nCr *x*^(*n* - *r*) y^r**. |
| **Fundamental Principle of Counting, Permutations & Combinations** | **Independent Term (Constant Term) Shortcuts** | In the expansion of (a*x*^*p* + *b* / *x*^*q*)^*n*, *the value of r for the term containing x^k is given directly by* ***r*** **= (*n*p - k) / (p + q)**. For the **term independent of x** (constant term), put k = 0, so **r = (n*p) / (p + q)**. |
| **Binomial Theorem for Positive Integral Index** | **If n is even, there is a single middle term** | the **(n/2 + 1)th term**, with binomial coefficient nC(n/2). |
| **Fundamental Principle of Counting, Permutations & Combinations** | **If n is odd, there are two middle terms** | the **((n + 1)/2)th** and **((n + 3)/2)th terms**, with equal binomial coefficients nC((n - 1)/2) = nC((n + 1)/2). |
| **Binomial Theorem for Positive Integral Index** | **Sum of all binomial coefficients** | **nC0 + nC1 + nC2 + ... + nCn = 2^n** (put x = 1, y = 1). |

## Chapter 4 Multi-Statement Exam Verification Drills

**Chapter 4 Statement-Verification Drill #1** — Both Statements Verified Correct

**Statement-I (Without repetition):** **nPr = n! / (n - r)!**.

**Statement-II (Circular Permutations):** The number of ways to arrange n distinct objects around a fixed circle is **(n - 1)!**. If clockwise and anticlockwise arrangements are indistinguishable (such as beads on a necklace or flowers in a garland), the number of arrangements is **(n - 1)! / 2** (for n >= 3).

✓ **Exam Verification Key: Both Statement-I and Statement-II are factually accurate as per UPSC Official Portal & IUC Academy.**

**Chapter 4 Statement-Verification Drill #2** — Both Statements Verified Correct

**Statement-I (Formula):** **nCr = n! / [r! * (n - r)!] = nPr / r!**.

**Statement-II (Symmetry Property):** **nCr = nC(n - r)**. Thus, if nCx = nCy, then either **x = y** or **x + y = n**.

✓ **Exam Verification Key: Both Statement-I and Statement-II are factually accurate as per UPSC Official Portal & IUC Academy.**

**Chapter 4 Statement-Verification Drill #3** — Both Statements Verified Correct

**Statement-I (Pascal's Identity):** **nCr + nC(r - 1) = (n + 1)Cr**.

**Statement-II (Ratio of consecutive combinations):** **nCr / nC(r - 1) = (n - r + 1) / r**.

✓ **Exam Verification Key: Both Statement-I and Statement-II are factually accurate as per UPSC Official Portal & IUC Academy.**

**Chapter 4 Statement-Verification Drill #4** — Both Statements Verified Correct

**Statement-I (Total ways of selecting one or more items from n distinct objects):** **2^n - 1 = nC1 + nC2 + ... + nCn**.

**Statement-II (Derangements (D_n)):** The number of ways to place n distinct letters into n addressed envelopes such that NO letter goes into its correct envelope is **D_n = n! * [1 - 1/1! + 1/2! - 1/3! + ... + (-1)^n / n!]**. Key values: D_1 = 0, D_2 = 1, D_3 = 2, D_4 = 9, D_5 = 44.

✓ **Exam Verification Key: Both Statement-I and Statement-II are factually accurate as per UPSC Official Portal & IUC Academy.**

**Chapter 4 Statement-Verification Drill #5** — Both Statements Verified Correct

**Statement-I (Total Number of Terms):** In the expansion of (x + y)^n, there are **(n + 1)** terms. More generally, in the multinomial expansion of (x1 + x2 + ... + xk)^n, the total number of distinct terms is **(n + k - 1)C(k - 1)**.

**Statement-II (General Term (r + 1)th Term):** **T_(r+1) = nCr $x^{(n - r)}$ y^r**.

✓ **Exam Verification Key: Both Statement-I and Statement-II are factually accurate as per UPSC Official Portal & IUC Academy.**

**Chapter 4 Statement-Verification Drill #6** — Both Statements Verified Correct

**Statement-I (Independent Term (Constant Term) Shortcuts):** In the expansion of *(ax^p + b / x^q)^n, the value of r for the term containing x^k is given directly by* **r = (np - k) / (p + q)**. For the **term independent of x** (constant term), put k = 0, so **r = (n*p) / (p + q)**.

**Statement-II (If n is even, there is a single middle term):** the **(n/2 + 1)th term**, with binomial coefficient nC(n/2).

✓ **Exam Verification Key: Both Statement-I and Statement-II are factually accurate as per UPSC Official Portal & IUC Academy.**

# Chapter 5: Limits, Continuity, Differentiability & Applications of Derivatives

**Chapter Key Study Pillars — Chapter 5: Limits, Continuity, Differentiability & Applications of Derivatives**

**Key Study Pillar 1**
**Evaluation of Limits & Indeterminate Forms**

**Key Study Pillar 2**
**Continuity and Differentiability**

**Key Study Pillar 3**
**Applications of Derivatives (AOD): Tangents, Monotonicity & Maxima/ Minima**

## 5.1 Evaluation of Limits & Indeterminate Forms

The limit of a function f(x) as x approaches a, written lim_(x->a) f(x) = L, exists if and only if the Left-Hand Limit (LHL = lim_(h->0) f(a - h)) and the Right-Hand Limit (RHL = lim_(h->0) f(a + h)) both exist, are finite, and are equal (LHL = RHL = L) (NCERT).

• **Seven Indeterminate Forms:** 0/0, infinity/infinity, 0 * infinity, infinity - infinity, 1^infinity, 0^0, and infinity^0.

• **Standard Fundamental Limits:**

• lim_(x->0) [sin(x) / x] = 1 and lim_(x->0) [tan(x) / x] = 1 (where x is strictly in radians).

• lim_(x->0) [(1 - cos(x)) / x^2] = 1/2.

• lim_(x->a) [(x^n - a^n) / (x - a)] = n * a^(n - 1).

• lim_(x->0) [(e^x - 1) / x] = 1 and lim_(x->0) [(a^x - 1) / x] = ln(a).

• lim_(x->0) [ln(1 + x) / x] = 1.

• **1^infinity Form Shortcut:** If lim_(x->a) f(x) = 1 and lim_(x->a) g(x) = infinity, then **lim_(x->a) [f(x)]^(g(x)) = e^(lim_(x->a) [g(x) (*f*(*x*) - 1)])**. *Consequently*, *lim_(x->0) (1 + kx)^(1/x) = e^k* and lim_(n->infinity) (1 + k/n)^n = e^k.

• **L'Hopital's Rule:** For 0/0 or infinity/infinity forms, lim_(x->a) [f(x) / g(x)] = lim_(x->a) [f'(x) / g'(x)], differentiating numerator and denominator separately until the indeterminate form vanishes.

## 5.2 Continuity and Differentiability

• **Continuity at a Point x = a:** A function f(x) is continuous at x = a if lim_(x->a-) f(x) = lim_(x->a+) f(x) = f(a).

• **Differentiability at x = a:** A function f(x) is differentiable at x = a if its Left-Hand Derivative (LHD = f'_(a-)) and Right-Hand Derivative (RHD = f'_(a+)) both exist finitely and are equal.

• **Core Theorem: Every differentiable function is necessarily continuous, but a continuous function need not be differentiable.** Classic counterexample: f(x) = |x - a| is continuous everywhere on R, but it is NOT differentiable at the sharp corner point x = a because LHD = -1 and RHD = +1 (UPSC).

• **Logarithmic, Implicit, and Parametric Differentiation:**

• For y = [u(x)]^[v(x)], take natural log on both sides or use: dy/dx = u^v [*v*' ln(u) + v * (u' / u)].

• For implicit equation F(x, y) = 0: **dy/dx = -(dF/dx) / (dF/dy)** (partial derivatives).

• For parametric curves x = f(t), y = g(t): dy/dx = (dy/dt) / (dx/dt), and **d^2y/dx^2 = [d/dt(dy/dx)] / (dx/dt)**.

## *5.3* Applications of Derivatives (AOD*)*: Tangents, Monotonicity & Maxima/Minima

• **Tangents and Normals:** At point P(x1, y1) on y = f(x), the slope of the tangent is m = (dy/dx)_(x1, y1), and the slope of the normal is -1/m.

• **Rolle's Theorem & Lagrange's Mean Value Theorem (LMVT):**

• **Rolle's Theorem:** If f(x) is continuous on [a, b], differentiable on (a, b), and f(a) = f(b), then there exists at least one c in (a, b) such that **f'(c) = 0**.

• **LMVT:** If f(x) is continuous on [a, b] and differentiable on (a, b), there exists at least one c in (a, b) such that **f'(c) = [f(b) - f(a)] / (b - a)**.

• **Monotonicity and Extrema (Maxima & Minima):**

• Increasing on I if f'(x) >= 0; Strictly Increasing if f'(x) > 0.

• **Second Derivative Test for Local Extrema:** Find critical points where f'(c) = 0. If **f''(c) < 0**, x = c is a point of **Local Maximum**; if **f''(c) > 0**, x = c is a point of **Local Minimum**; if f''(c) = 0 and f'''(c) != 0, x = c is a **Point of Inflection**.

**Step-by-Step Conceptual Process Flowchart**

**STEP 1** Check Limit Existence (LHL = RHL)

**STEP 2** Verify Continuity (Limit = f(a))

**STEP 3** Test Differentiability (LHD = RHD)

**STEP 4** Set f'(x) = 0 and Check Sign of f''(x) for Extrema

| Function / Scenario | Critical Point(s) | Continuity & Differentiability Status | Extrema / Special Value | Official Reference | | |
|---|---|---|---|---|---|---|
| **f(x) = \|x\|** | x = 0 | Continuous everywhere; Not differentiable at x = 0 | Global Minimum = 0 at x = 0 | NCERT Class XII | | |
| **f(x) = [x] (Greatest Integer)** | All integers x in Z | Discontinuous and Non-differentiable at all integers Z | Step function; f'(x) = 0 for non-integers | UPSC NDA Portal | | |
| **f(x) = x^(1/x) (x > 0)** | x = e | Continuous and Differentiable for all x > 0 | Global Maximum = e^(1/e) at x = e | UPSC NDA Portal | | |
| **f(x) = sin(x) + cos(x)** | x = pi/4, 5*pi/4 | Continuous and Differentiable everywhere on R | Max = +sqrt(2), Min = -sqrt(2) | IUC Academy | | |
| **f(x) = *a*sin(*x*) + *b*cos(x)** | tan(x) = a/b | Smooth periodic function with period 2*pi | Max = +sqrt(a^2+b^2), Min = -sqrt(a^2+b^2) | IUC Academy | | |

**Did You Know?:** For the function f(x) = x^(1/x) (or ln(x)/x), the global maximum always occurs at x = e (approx 2.718). Because 2 < e < 3 and f(x) is strictly decreasing for x > e, this immediately proves without calculation that e^pi > pi^e and 3^(1/3) > 4^(1/4) > 5^(1/5) (IUC Academy).

**Exam Trap Alert:** When computing lim_(x->0) [sin(x degrees) / x], the answer is NOT 1! Because 1 degree = pi / 180 radians, sin(x degrees) = sin(pi*x / 180), so lim_(x->0) [sin(x degrees) / x] = **pi / 180** (UPSC).

★ **Memory Trick:** For Second Derivative Maxima/Minima, remember **"Negative is a Peak, Positive is a Valley"**: when f''(c) < 0 (Negative), the curve is concave down like an inverted umbrella, forming a **Maximum Peak**; when f''(c) > 0 (Positive), the curve is concave up like a bowl, forming a **Minimum Valley** (NCERT).

## Chapter 5 Comprehensive Analytical & Statutory Revision Matrix

| Core Sub-Domain | Key Concept / Provision | Verified Analytical Parameters & Exam Distinctions (UPSC Official Portal) |
|---|---|---|
| **Evaluation of Limits & Indeterminate Forms** | **Seven Indeterminate Forms** | 0/0, infinity/infinity, 0 * infinity, infinity - infinity, 1^infinity, 0^0, and infinity^0. |
| **Continuity and Differentiability** | **1^infinity Form Shortcut** | If lim_(x->a) f(x) = 1 and lim_(x->a) g(x) = infinity, then **lim_(x->a) [f(x)]^(g(x)) = e^(lim_(x->a) [g(x) (*f*(*x*) - 1)])**. *Consequently*, *lim_(x->0) (1 + kx)^(1/x) = e^k* and lim_(n->infinity) (1 + k/n)^n = e^k. |
| **Applications of Derivatives (AOD): Tangents, Monotonicity & Maxima/Minima** | **L'Hopital's Rule** | For 0/0 or infinity/infinity forms, lim_(x->a) [f(x) / g(x)] = lim_(x->a) [f'(x) / g'(x)], differentiating numerator and denominator separately until the indeterminate form vanishes. |
| **Evaluation of Limits & Indeterminate Forms** | **Continuity at a Point x = a** | A function f(x) is continuous at x = a if lim_(x->a-) f(x) = lim_(x->a+) f(x) = f(a). |
| **Continuity and Differentiability** | **Differentiability at x = a** | A function f(x) is differentiable at x = a if its Left-Hand Derivative (LHD = f'_(a-)) and Right-Hand Derivative (RHD = f'_(a+)) both exist finitely and are equal. |
| **Applications of Derivatives (AOD): Tangents, Monotonicity & Maxima/Minima** | **Core Theorem** | **Every differentiable function is necessarily continuous, but a continuous function need not be differentiable.** Classic counterexample: f(x) = \|x - a\| is continuous everywhere on R, but it is NOT differentiable at the sharp corner point x = a because LHD = -1 and RHD = +1 (UPSC). |
| **Evaluation of Limits & Indeterminate Forms** | **For y = [u(x)]^[v(x)], take natural log on both sides or use** | dy/dx = u^v [*v*' ln(u) + v * (u' / u)]. |
| **Continuity and Differentiability** | **For implicit equation F(x, y) = 0** | **dy/dx = -(dF/dx) / (dF/dy)** (partial derivatives). |
| **Applications of Derivatives (AOD): Tangents, Monotonicity & Maxima/Minima** | **For parametric curves x = f(t), y = g(t)** | dy/dx = (dy/dt) / (dx/dt), and **d^2y/dx^2 = [d/dt(dy/dx)] / (dx/dt)**. |

| Core Sub-Domain | Key Concept / Provision | Verified Analytical Parameters & Exam Distinctions (UPSC Official Portal) |
|---|---|---|
| **Evaluation of Limits & Indeterminate Forms** | **Tangents and Normals** | At point P(x1, y1) on y = f(x), the slope of the tangent is m = (dy/dx)_(x1, y1), and the slope of the normal is -1/m. |
| **Continuity and Differentiability** | **Rolle's Theorem** | If f(x) is continuous on [a, b], differentiable on (a, b), and f(a) = f(b), then there exists at least one c in (a, b) such that **f'(c) = 0**. |
| **Applications of Derivatives (AOD): Tangents, Monotonicity & Maxima/Minima** | **LMVT** | If f(x) is continuous on [a, b] and differentiable on (a, b), there exists at least one c in (a, b) such that **f'(c) = [f(b) - f(a)] / (b - a)**. |
| **Evaluation of Limits & Indeterminate Forms** | **Second Derivative Test for Local Extrema** | Find critical points where f'(c) = 0. If **f''(c) < 0**, x = c is a point of **Local Maximum**; if **f''(c) > 0**, x = c is a point of **Local Minimum**; if f''(c) = 0 and f'''(c) != 0, x = c is a **Point of Inflection**. |

## Chapter *5* Multi-Statement Exam Verification Drills

**Chapter 5 Statement-Verification Drill #1** — Both Statements Verified Correct

**Statement-I (Seven Indeterminate Forms):** 0/0, infinity/infinity, 0 * infinity, infinity - infinity, 1^infinity, 0^0, and infinity^0.
**Statement-II (1^infinity Form Shortcut):** If lim_(x->a) f(x) = 1 and lim_(x->a) g(x) = infinity, then **lim_(x->a) [f(x)]^(g(x)) = e^(lim_(x->a) [g(x) (*f*(*x*) - 1)])**. *Consequently*, *lim_*(*x*->0) (1 + *k*x)^(1/x) = e^k and lim_(n->infinity) (1 + k/n)^n = e^k.

✓ **Exam Verification Key: Both Statement-I and Statement-II are factually accurate as per UPSC Official Portal & IUC Academy.**

**Chapter 5 Statement-Verification Drill #2** — Both Statements Verified Correct

**Statement-I (L'Hopital's Rule):** For 0/0 or infinity/infinity forms, lim_(x->a) [f(x) / g(x)] = lim_(x->a) [f'(x) / g'(x)], differentiating numerator and denominator separately until the indeterminate form vanishes.
**Statement-II (Continuity at a Point x = a):** A function f(x) is continuous at x = a if lim_(x->a-) f(x) = lim_(x->a+) f(x) = f(a).

✓ **Exam Verification Key: Both Statement-I and Statement-II are factually accurate as per UPSC Official Portal & IUC Academy.**

**Chapter 5 Statement-Verification Drill #3** — Both Statements Verified Correct

**Statement-I (Differentiability at x = a):** A function f(x) is differentiable at x = a if its Left-Hand Derivative (LHD = f'_(a-)) and Right-Hand Derivative (RHD = f'_(a+)) both exist finitely and are equal.
**Statement-II (Core Theorem): Every differentiable function is necessarily continuous, but a continuous function need not be differentiable.** Classic counterexample: f(x) = |x - a| is continuous everywhere on R, but it is NOT differentiable at the sharp corner point x = a because LHD = -1 and RHD = +1 (**UPSC**).

✓ **Exam Verification Key: Both Statement-I and Statement-II are factually accurate as per UPSC Official Portal & IUC Academy.**

**Chapter 5 Statement-Verification Drill #4** — Both Statements Verified Correct

**Statement-I (For y = [u(x)]^[v(x)], take natural log on both sides or use):** dy/dx = u^v [v' ln(u) + v * (u' / u)].
**Statement-II (For implicit equation F(x, y) = 0): dy/dx = -(dF/dx) / (dF/dy)** (partial derivatives).

✓ **Exam Verification Key: Both Statement-I and Statement-II are factually accurate as per UPSC Official Portal & IUC Academy.**

**Chapter 5 Statement-Verification Drill #5** — Both Statements Verified Correct

**Statement-I (For parametric curves x = f(t), y = g(t)):** dy/dx = (dy/dt) / (dx/dt), and **d^2y/dx^2 = [d/dt(dy/dx)] / (dx/dt)**.

**Statement-II (Tangents and Normals):** At point P(x1, y1) on y = f(x), the slope of the tangent is m = (dy/dx)_(x1, y1), and the slope of the normal is -1/m.

✓ **Exam Verification Key: Both Statement-I and Statement-II are factually accurate as per UPSC Official Portal & IUC Academy.**

**Chapter 5 Statement-Verification Drill #6** — Both Statements Verified Correct

**Statement-I (Rolle's Theorem):** If f(x) is continuous on [a, b], differentiable on (a, b), and f(a) = f(b), then there exists at least one c in (a, b) such that **f'(c) = 0**.

**Statement-II (LMVT):** If f(x) is continuous on [a, b] and differentiable on (a, b), there exists at least one c in (a, b) such that **f'(c) = [f(b) - f(a)] / (b - a)**.

✓ **Exam Verification Key: Both Statement-I and Statement-II are factually accurate as per UPSC Official Portal & IUC Academy.**

# Chapter 6: Indefinite & Definite Integration, Area Under Curves & Differential Equations

**Chapter Key Study Pillars — Chapter 6: Indefinite & Definite Integration, Area Under Curves & Differential Equations**

| Key Study Pillar 1 | Key Study Pillar 2 | Key Study Pillar 3 |
|---|---|---|
| **Indefinite Integration & Special Integrals** | **Definite Integration Properties (King's Rule & Queen's Rule)** | **Area Under Curves & Differential Equations** |

## 6.1 Indefinite Integration & Special Integrals

Integration is the inverse process of differentiation: if d/dx [F(x)] = f(x), then integral f(x) dx = F(x) + C, where C is the arbitrary constant of integration (NCERT).

- **Standard Integration Methods & Classic Forms:**
- **Substitution Form:** integral [f'(x) / f(x)] dx = ln|f(x)| + C, and integral [f(x)]^n * f'(x) dx = [f(x)]^(n + 1) / (n + 1) + C (for n != -1).
- **Integration by Parts (ILATE Rule):** integral (u v) dx = u (integral v dx) - integral [(du/dx) * (integral v dx)] dx, where the first function u is chosen according to the priority order **I**nverse Trig, **L**ogarithmic, **A**lgebraic, **T**rigonometric, **E**xponential.
- **Exponential Cancellation Formula: integral e^x [f(x) + f'(x)] dx = e^x f(x) + C.**
- **Nine Standard Quadratic Integrals:**
- integral dx / (x^2 + a^2) = (1/a) * arctan(x/a) + C
- integral dx / (x^2 - a^2) = (1/(2a)) ln|(x - a)/(x + a)| + C
- integral dx / (a^2 - x^2) = (1/(2a)) ln|(a + x)/(a - x)| + C
- integral dx / sqrt(a^2 - x^2) = arcsin(x/a) + C
- integral dx / sqrt(x^2 +/- a^2) = ln|x + sqrt(x^2 +/- a^2)| + C

## *6.2* Definite Integration Properties (King's Rule & Queen's Rule)

By the Fundamental Theorem of Calculus, integral_a^b f(x) dx = F(b) - F(a). Competitive exams such as **UPSC NDA** rely heavily on symmetry properties rather than brute-force antiderivatives.

• **Key Properties of Definite Integrals:**

• **Property P1 (Interval Reversal):** integral_a^b f(x) dx = - integral_b^a f(x) dx, and integral_a^a f(x) dx = 0.

• **Property P2 (Piecewise Splitting for Modulus/GIF):** integral_a^b f(x) dx = integral_a^c f(x) dx + integral_c^b f(x) dx.

• **Property P3 (King's Property): integral_a^b f(x) dx = integral_a^b f(a + b - x) dx**. Specifically, for [0, a]: **integral_0^a f(x) dx = integral_0^a f(a - x) dx**.

• **High-Speed Symmetry Corollary:** For any function g(x), **integral_a^b [g(x) / (g(x) + g(a + b - x))] dx = (b - a) / 2**.

• **Property P4 (Even-Odd Rule on [-a, a]):** integral_(-a)^a f(x) dx = **0** if f(x) is an **odd function** (f(-x) = -f(x)), and equals **2 * integral_0^a f(x) dx** if f(x) is an **even function** (f(-x) = f(x)).

• **Property P5 (Queen's Property on [0, 2a]):** integral_0^(2a) f(x) dx = 2 * integral_0^a f(x) dx if f(2a - x) = f(x), and equals 0 if f(2a - x) = -f(x).

• **Wallis' Formula:** integral_0^(pi/2) sin^n(x) dx = integral_0^(pi/2) cos^n(x) dx = [(n - 1)(*n* - 3)...(1 *or* 2)] / [*n*(n - 2)...(1 or 2)] * K, where K = pi/2 if n is even, and K = 1 if n is odd.

• **Leibniz Rule (Differentiation Under Integral Sign):** d/dx [integral_(g(x))^(h(x)) f(t) dt] = f(h(x)) *h*'(*x*) - *f*(*g*(*x*)) g'(x).

## *6.3* Area Under Curves & Differential Equations

• **Standard Area Formulas (QuadratureShortcuts):**

• Area bounded by parabola y^2 = 4*a*x and its latus rectum x = a is **8/3 * a^2**.

• Area bounded by parabola y^2 = 4*a*x and line y = m*x is* **8 a^2 / (3 * m^3)**.

• Area bounded between two intersecting parabolas y^2 = 4*a*x and x^2 = 4*b*y is **16 *a* b / 3**.

• Area of the full ellipse x^2/a^2 + y^2/b^2 = 1 is **pi *a* b**.

• **Differential Equations (Order, Degree & Solution Methods):**

• **Order:** The order of the highest-order derivative present in the differential equation.

• **Degree:** The highest power (exponent) of the highest-order derivative AFTER the equation has been made free from radicals and fractional powers of all derivatives (and derivatives are not trapped inside transcendental functions like sin(dy/dx) or e^(dy/dx)).

• **First-Order Linear Differential Equation:** Form **dy/dx + P(x)*y* = *Q*(*x*)**. *The **Integrating Factor (IF)** = **e^(*integral P*(*x*) *dx*)**, and the general solution is **y (IF) = integral [Q(x) * (IF)] dx + C**.

**Step-by-Step Conceptual Process Flowchart**

**STEP 1**
Clear Radicals/Fractions to Identify Order & Degree

**STEP 2**
Apply King's Rule or Even-Odd Symmetry to Definite Integrals

**STEP 3**
Compute Integrating Factor IF = e^(integral P dx) for Linear DE

**STEP 4**
Apply Shortcut Formula for Parabola/Ellipse Bounded Area

| Definite Integral / Geometric Region | Exact Closed-Form Result | Condition / Parameters | Official Source |
|---|---|---|---|
| **integral_a^b [f(x) / (f(x) + f(a + b - x))] dx** | (b - a) / 2 | Valid for any continuous f(x) | UPSC NDA Portal |
| **integral_0^(pi/2) ln(sin(x)) dx = integral_0^(pi/2) ln(cos(x)) dx** | -(pi / 2) * ln(2) | Classic Euler log-sine integral | NCERT Class XII |
| **Area between y^2 = 4*a*x and x^2 = 4*b*y** | (16 *a* b) / 3 sq. units | Intersecting orthogonal parabolas | UPSC NDA Portal |
| **Area between y^2 = 4*a*x and line y = m*x** | (8 *a*^2) / (3 m^3) sq. units | Parabola and chord through vertex | IUC Academy |
| **Degree of d^2y/dx^2 + sin(dy/dx) = 0** | Order = 2, Degree = Not Defined | dy/dx is inside sin() | NCERT Class XII |

**Did You Know?:** Every integral of the form integral_0^(pi/2) [sin^n(x) / (sin^n(x) + cos^n(x))] dx, regardless of whether the power n is 1, 2, 1/2, 2026, or -5, always evaluates to the exact same value: **pi / 4**, directly by applying King's Rule (x -> pi/2 - x) (NCERT).

**Exam Trap Alert:** The **Order** of a differential equation is ALWAYS a defined positive integer, but the **Degree** is **NOT DEFINED** whenever any derivative (such as dy/dx or d^2y/dx^2) is trapped inside a trigonometric, logarithmic, or exponential function like sin(dy/dx), ln(dy/dx), or e^(dy/dx) (UPSC).

★ **Memory Trick:** For parabola area questions, remember the **"16-8-8 Rule of 3"**: Two parabolas y^2 = 4ax and x^2 = 4by enclose **(16*a*b)/3**; one parabola y^2 = 4ax with its latus rectum encloses **(8*a*^2)/3**; *and one parabola y^2 = 4ax with line y = mx encloses* **(8a^2)/(3*m^3)** (IUC Academy).

## Chapter *6* Comprehensive Analytical & Statutory Revision Matrix

| Core Sub-Domain | Key Concept / Provision | Verified Analytical Parameters & Exam Distinctions (UPSC Official Portal) |
|---|---|---|
| **Indefinite Integration & Special Integrals** | **Substitution Form** | integral [f'(x) / f(x)] dx = ln \|f(x)\| + C, and integral [f(x)]^n * f'(x) dx = [f(x)]^(n + 1) / (n + 1) + C (for n != -1). |

| Core Sub-Domain | Key Concept / Provision | Verified Analytical Parameters & Exam Distinctions (UPSC Official Portal) |
|---|---|---|
| **Definite Integration Properties (King's Rule & Queen's Rule)** | **Integration by Parts (ILATE Rule)** | integral (u *v*) *dx* = *u* (integral v dx) - integral [(du/dx) * (integral v dx)] dx, where the first function u is chosen according to the priority order **I**nverse Trig, **L**ogarithmic, **A**lgebraic, **T**rigonometric, **E**xponential. |
| **Area Under Curves & Differential Equations** | **Exponential Cancellation Formula** | **integral e^x [*f(x)* + *f'(x)*] *dx* = *e^x* f(x) + C**. |
| **Indefinite Integration & Special Integrals** | **Property P1 (Interval Reversal)** | integral_a^b f(x) dx = - integral_b^a f(x) dx, and integral_a^a f(x) dx = 0. |
| **Definite Integration Properties (King's Rule & Queen's Rule)** | **Property P2 (Piecewise Splitting for Modulus/GIF)** | integral_a^b f(x) dx = integral_a^c f(x) dx + integral_c^b f(x) dx. |
| **Area Under Curves & Differential Equations** | **Property P3 (King's Property)** | **integral_a^b f(x) dx = integral_a^b f(a + b - x) dx**. Specifically, for *[0, a]*: **integral_0^a f(x) dx = integral_0^a f(a - x) dx**. |
| **Indefinite Integration & Special Integrals** | **High-Speed Symmetry Corollary** | For any function g(x), **integral_a^b [g(x) / (g(x) + g(a + b - x))] dx = (b - a) / 2**. |
| **Definite Integration Properties (King's Rule & Queen's Rule)** | **Property P4 (Even-Odd Rule on [-a, a])** | integral_(-a)^a f(x) dx = **0** if f(x) is an **odd function** (f(-x) = -f(x)), and equals **2 * integral_0^a f(x) dx** if f(x) is an **even function** (f(-x) = f(x)). |
| **Area Under Curves & Differential Equations** | **Property P5 (Queen's Property on [0, 2a])** | integral_0^(2a) f(x) dx = 2 * integral_0^a f(x) dx if f(2a - x) = f(x), and equals 0 if f(2a - x) = -f(x). |
| **Indefinite Integration & Special Integrals** | **Wallis' Formula** | integral_0^(pi/2) sin^n(x) dx = integral_0^(pi/2) cos^n(x) dx = *[(n - 1)(n - 3)... (1 or 2)] / [n*(n - 2)...(1 or 2)] * K, where K = pi/2 if n is even, and K = 1 if n is odd. |
| **Definite Integration Properties (King's Rule & Queen's Rule)** | **Leibniz Rule (Differentiation Under Integral Sign)** | d/dx [integral_(g(x))^(h(x)) f(t) dt] = f(h(x)) *h'(x) - f(g(x))* g'(x). |
| **Area Under Curves & Differential Equations** | **Order** | The order of the highest-order derivative present in the differential equation. |
| **Indefinite Integration & Special Integrals** | **Degree** | The highest power (exponent) of the highest-order derivative AFTER the equation has been made free from radicals and fractional powers of all derivatives (and derivatives are not trapped inside transcendental functions like sin(dy/dx) or e^(dy/dx)). |

| Core Sub-Domain | Key Concept / Provision | Verified Analytical Parameters & Exam Distinctions (UPSC Official Portal) |
|---|---|---|
| **Definite Integration Properties (King's Rule & Queen's Rule)** | **First-Order Linear Differential Equation** | Form **dy/dx + P(x)*y* = *Q*(*x*)**. *The* ***Integrating Factor (IF)*** *=* ***e^(integral P(x) dx)****, and the general solution is* ***y*** **(IF) = integral [Q(x) * (IF)] dx + C**. |

## Chapter *6* Multi-Statement Exam Verification Drills

**Chapter 6 Statement-Verification Drill #1** — Both Statements Verified Correct

**Statement-I (Substitution Form):** integral [f'(x) / f(x)] dx = ln|f(x)| + C, and integral [f(x)]^n * f'(x) dx = [f(x)]^(n + 1) / (n + 1) + C (for n != -1).

**Statement-II (Integration by Parts (ILATE Rule)):** integral (u *v*) *dx* = *u* (integral v dx) - integral [(du/dx) * (integral v dx)] dx, where the first function u is chosen according to the priority order **I**nverse Trig, **L**ogarithmic, **A**lgebraic, **T**rigonometric, **E**xponential.

✓ **Exam Verification Key: Both Statement-I and Statement-II are factually accurate as per UPSC Official Portal & IUC Academy.**

**Chapter 6 Statement-Verification Drill #2** — Both Statements Verified Correct

**Statement-I (Exponential Cancellation Formula): integral e^x [*f*(*x*) + *f*'(*x*)] *dx* = *e*^*x* f(x) + C.**

**Statement-II (Property P1 (Interval Reversal)):** integral_a^b f(x) dx = - integral_b^a f(x) dx, and integral_a^a f(x) dx = 0.

✓ **Exam Verification Key: Both Statement-I and Statement-II are factually accurate as per UPSC Official Portal & IUC Academy.**

**Chapter 6 Statement-Verification Drill #3** — Both Statements Verified Correct

**Statement-I (Property P2 (Piecewise Splitting for Modulus/GIF)):** integral_a^b f(x) dx = integral_a^c f(x) dx + integral_c^b f(x) dx.

**Statement-II (Property P3 (King's Property)): integral_a^b f(x) dx = integral_a^b f(a + b - x) dx**. Specifically, for [0, a]: **integral_0^a f(x) dx = integral_0^a f(a - x) dx**.

✓ **Exam Verification Key: Both Statement-I and Statement-II are factually accurate as per UPSC Official Portal & IUC Academy.**

**Chapter 6 Statement-Verification Drill #4** — Both Statements Verified Correct

**Statement-I (High-Speed Symmetry Corollary):** For any function g(x), **integral_a^b [g(x) / (g(x) + g(a + b - x))] dx = (b - a) / 2**.

**Statement-II (Property P4 (Even-Odd Rule on [-a, a])):** integral_(-a)^a f(x) dx = **0** if f(x) is an **odd function** (f(-x) = -f(x)), and equals **2 * integral_0^a f(x) dx** if f(x) is an **even function** (f(-x) = f(x)).

✓ **Exam Verification Key: Both Statement-I and Statement-II are factually accurate as per UPSC Official Portal & IUC Academy.**

**Chapter 6 Statement-Verification Drill #5** — Both Statements Verified Correct

**Statement-I (Property P5 (Queen's Property on [0, 2a])):** integral_0^(2a) f(x) dx = 2 * integral_0^a f(x) dx if f(2a - x) = f(x), and equals 0 if f(2a - x) = -f(x).

**Statement-II (Wallis' Formula):** integral_0^(pi/2) sin^n(x) dx = integral_0^(pi/2) cos^n(x) dx = [(n - 1)(*n* - 3)...(1 *or* 2)] / [*n*(n - 2)...(1 or 2)] * K, where K = pi/2 if n is even, and K = 1 if n is odd.

✓ **Exam Verification Key: Both Statement-I and Statement-II are factually accurate as per UPSC Official Portal & IUC Academy.**

**Chapter 6 Statement-Verification Drill #6** — Both Statements Verified Correct

**Statement-I (Leibniz Rule (Differentiation Under Integral Sign)):** d/dx [integral_(g(x))^(h(x)) f(t) dt] = f(h(x)) *h'(x) - f(g(x))* g'(x).

**Statement-II (Order):** The order of the highest-order derivative present in the differential equation.

✓ **Exam Verification Key: Both Statement-I and Statement-II are factually accurate as per UPSC Official Portal & IUC Academy.**

# Chapter 7: Vector Algebra & Three-Dimensional (3D) Coordinate Geometry

**Chapter Key Study Pillars — Chapter 7: Vector Algebra & Three-Dimensional (3D) Coordinate Geometry**

**Key Study Pillar 1**
**Vector Algebra: Dot Product, Cross Product & Scalar Triple Product**

**Key Study Pillar 2**
**Three-Dimensional (3D) Geometry: Lines, Planes & Spheres**

## 7.1 Vector Algebra: Dot Product, Cross Product & Scalar Triple Product

A vector possesses both magnitude and direction. If vector r = x *i_hat* + yj_hat + z*k_hat, its magnitude is |r| = sqrt(x^2 + y^2 + z^2), and the unit vector along r is r_hat = r / |r| (**NCERT**).

- **Direction Cosines (l, m, n) and Direction Ratios (a, b, c):**
- If a vector makes angles alpha, beta, gamma with the positive X, Y, Z axes, its direction cosines are **l = cos(alpha), m = cos(beta), n = cos(gamma)**.
- Fundamental Identity: **l^2 + m^2 + n^2 = cos^2(alpha) + cos^2(beta) + cos^2(gamma) = 1**, which implies **sin^2(alpha) + sin^2(beta) + sin^2(gamma) = 2**.
- **Scalar (Dot) Product (a . b):**
- Defined as **a . b = |a| |b| cos(theta) = a1*b1 + a2b2 + a3*b3**.
- **Orthogonality Condition:** Two non-zero vectors a and b are perpendicular if and only if **a . b = 0**.
- **Scalar Projection of vector a on vector b: (a . b) / |b|.**
- **Vector (Cross) Product (a x b):**
- Defined as **a x b = |a| |b| sin(theta) * n_hat**, where n_hat is a unit vector perpendicular to both a and b.
- **Collinearity / Parallelism Condition:** a x b = 0 (components are proportional: a1/b1 = a2/b2 = a3/b3).
- **Geometric Area Applications:** Area of a parallelogram with adjacent sides a and b is **|a x b|**. Area of a parallelogram with diagonals d1 and d2 is **(1/2) |d1 x d2|**. *Area of a triangle with adjacent sides a and b is* **(1/2) |a x b|**.
- **Lagrange's Identity: |a x b|^2 + (a . b)^2 = |a|^2 * |b|^2.**
- **Scalar Triple Product (STP / Box Product [a b c]):**
- Defined as **[a b c] = a . (b x c)**, computed as the 3 x 3 determinant of the components of a, b, and c, representing the volume of a parallelepiped with coterminous edges a, b, and c.
- **Coplanarity Condition:** Three vectors a, b, c are coplanar if and only if **[a b c] = 0**.

## 7.2 Three-Dimensional (3D) Geometry: Lines, Planes & Spheres

Three-dimensional geometry is tested thoroughly in UPSC NDA and IUC Academy practice papers.

- **Equation of a Straight Line in 3D Space:**
- Passing through point A(x1, y1, z1) with direction ratios (a, b, c): **(x - x1)/a = (y - y1)/b = (z - z1)/c = lambda** (Vector form: r = a_vec + lambda * b_vec).
- **Shortest Distance between Two Skew Lines** r = a1 + lambda*b1 and r = a2 + mu*b2: **SD = |(a2 - a1) . (b1 x b2)| / |b1 x b2|**. Two lines are coplanar (intersecting or parallel) if and only if (a2 - a1) . (b1 x b2) = 0.
- **Equation of a Plane & Distance Formulas:**
- General Cartesian Form: **Ax + By + C*z + D = 0**, where (A, B, C) are direction ratios of the normal vector to the plane.
- Intercept Form: **x/a + y/b + z/c = 1**.
- Perpendicular distance from point P(x1, y1, z1) to plane Ax + By + Cz + D = 0: **d = |Ax1 + By1 + Cz1 + D| / sqrt(A^2 + B^2 + C^2)**.
- Distance between two parallel planes Ax + By + Cz + D1 = 0 and Ax + By + Cz + D2 = 0: **d = |D1 - D2| / sqrt(A^2 + B^2 + C^2)**.
- **Image / Foot of Perpendicular (x2, y2, z2) of Point (x1, y1, z1) in Plane Ax + By + C*z + D = 0:** **(x2 - x1)/A = (y2 - y1)/B = (z2 - z1)/C = -k (Ax1 + By1 + Cz1 + D) / (A^2 + B^2 + C^2)**, where **k = 1 for Foot of Perpendicular** and **k = 2 for Mirror Image**.
- **Equation of a Sphere:** General equation is **x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0**, with **Center = (-u, -v, -w)** and **Radius R = sqrt(u^2 + v^2 + w^2 - d)**.

**Step-by-Step Conceptual Process Flowchart**

**STEP 1** Extract Position Vectors & Direction Ratios (a, b, c)

**STEP 2** Apply Dot Product (Orthogonality) or Cross Product (Normal Vector)

**STEP 3** Formulate Cartesian/ Vector Equation of Line or Plane

**STEP 4** Compute Shortest Distance, Foot of Perpendicular, or Image

| 3D Geometric Configuration | Formula / Condition | Key Geometric Note | Official Reference |
|---|---|---|---|
| **Direction Cosines Identity** | l^2 + m^2 + n^2 = 1; sum(sin^2) = 2 | cos(2alpha) + cos(2beta) + cos(2*gamma) = -1 | NCERT Class XII |
| **Angle between Line (b_vec) and Plane (n_vec)** | sin(theta) = | b_vec . n_vec | / ( | b_vec | * | n_vec | ) | Uses sin(theta) because normal is perpendicular to plane | UPSC NDA Portal |

| 3D Geometric Configuration | Formula / Condition | Key Geometric Note | Official Reference | | | |
|---|---|---|---|---|---|---|
| **Shortest Distance Between Parallel Lines** | d = | b_vec x (a2 - a1) | / | b_vec | | Cross product divided by direction magnitude | NCERT Class XII |
| **Volume of Tetrahedron (Coterminous Edges a, b, c)** | V = (1/6) * | [a b c] | | 1/6th the volume of corresponding parallelepiped | UPSC NDA Portal | |
| **Sphere with Diameter Endpoints (x1,y1,z1) & (x2,y2,z2)** | (x-x1)(x-x2) + (y-y1)(y-y2) + (z-z1)(z-z2) = 0 | Direct diameter form of sphere | IUC Academy | | | |

**Did You Know?:** A plane cuts a sphere in a circle! If R is the radius of the sphere and p is the perpendicular distance from the center of the sphere to the intersecting plane, the radius r of the circular cross-section is always given by the Pythagorean relation **r = sqrt(R^2 - p^2)** (UPSC).

**Exam Trap Alert:** When finding the angle theta between two lines or two planes, you use **cos(theta)**; however, when finding the angle theta between a **Line and a Plane**, you MUST use **sin(theta) = |b . n| / (|b| * |n|)** because the plane's equation gives its normal vector (90 degrees to the plane) rather than a vector lying in the plane (NCERT).

★ **Memory Trick:** Remember the **"1-2 Mirror Formula"** for a point (x1, y1, z1) and plane Ax + By + Cz + D = 0: the multiplier in -(k)*(Ax1 + By1 + Cz1 + D)/(A^2 + B^2 + C^2) is **k = 1 for the Foot of the Perpendicular** (halfway to the mirror image) and **k = 2 for the Mirror Image** (IUC Academy).

## Chapter 7 Comprehensive Analytical & Statutory Revision Matrix

| Core Sub-Domain | Key Concept / Provision | Verified Analytical Parameters & Exam Distinctions (UPSC Official Portal) |
|---|---|---|
| **Vector Algebra: Dot Product, Cross Product & Scalar Triple Product** | **Fundamental Identity** | **l^2 + m^2 + n^2 = cos^2(alpha) + cos^2(beta) + cos^2(gamma) = 1**, which implies **sin^2(alpha) + sin^2(beta) + sin^2(gamma) = 2**. |
| **Three-Dimensional (3D) Geometry: Lines, Planes & Spheres** | **Orthogonality Condition** | Two non-zero vectors a and b are perpendicular if and only if **a . b = 0**. |

| Core Sub-Domain | Key Concept / Provision | Verified Analytical Parameters & Exam Distinctions (UPSC Official Portal) |
|---|---|---|
| **Vector Algebra: Dot Product, Cross Product & Scalar Triple Product** | **Collinearity / Parallelism Condition** | a x b = 0 (components are proportional: a1/b1 = a2/b2 = a3/b3). |
| **Three-Dimensional (3D) Geometry: Lines, Planes & Spheres** | **Geometric Area Applications** | Area of a parallelogram with adjacent sides a and b is **\|a x b\|**. Area of a parallelogram with diagonals d1 and d2 is **(1/2) \|*d1 x d2*\|**. *Area of a triangle with adjacent sides a and b is* **(1/2) \|a x b\|**. |
| **Vector Algebra: Dot Product, Cross Product & Scalar Triple Product** | **Lagrange's Identity** | **\|a x b\|^2 + (a . b)^2 = \|a\|^2 * \|b\|^2**. |
| **Three-Dimensional (3D) Geometry: Lines, Planes & Spheres** | **Coplanarity Condition** | Three vectors a, b, c are coplanar if and only if **[a b c] = 0**. |
| **Vector Algebra: Dot Product, Cross Product & Scalar Triple Product** | **Passing through point A(x1, y1, z1) with direction ratios (a, b, c)** | **(x - x1)/a = (y - y1)/b = (z - z1)/c = lambda** (Vector form: r = a_vec + lambda * b_vec). |
| **Three-Dimensional (3D) Geometry: Lines, Planes & Spheres** | **Shortest Distance between Two Skew Lines** | r = a1 + lambda*b1 and r = a2 + mu*b2: **SD = \|(a2 - a1) . (b1 x b2)\| / \|b1 x b2\|**. Two lines are coplanar (intersecting or parallel) if and only if (a2 - a1) . (b1 x b2) = 0. |
| **Vector Algebra: Dot Product, Cross Product & Scalar Triple Product** | **General Cartesian Form** | **A*x* + *B*y + C*z + D = 0**, where (A, B, C) are direction ratios of the normal vector to the plane. |
| **Three-Dimensional (3D) Geometry: Lines, Planes & Spheres** | **Intercept Form** | **x/a + y/b + z/c = 1**. |
| **Vector Algebra: Dot Product, Cross Product & Scalar Triple Product** | **Equation of a Sphere** | General equation is **x^2 + y^2 + z^2 + 2*u*x + 2*v*y + 2*w*z + d = 0**, with **Center = (-u, -v, -w)** and **Radius R = sqrt(u^2 + v^2 + w^2 - d)**. |

## Chapter 7 Multi-Statement Exam Verification Drills

**Chapter 7 Statement-Verification Drill #1** — Both Statements Verified Correct

**Statement-I (Fundamental Identity):** l^2 + m^2 + n^2 = cos^2(alpha) + cos^2(beta) + cos^2(gamma) = 1, which implies sin^2(alpha) + sin^2(beta) + sin^2(gamma) = 2.

**Statement-II (Orthogonality Condition):** Two non-zero vectors a and b are perpendicular if and only if **a . b = 0**.

✓ **Exam Verification Key: Both Statement-I and Statement-II are factually accurate as per UPSC Official Portal & IUC Academy.**

**Chapter 7 Statement-Verification Drill #2** — Both Statements Verified Correct

**Statement-I (Collinearity / Parallelism Condition):** a x b = 0 (components are proportional: a1/b1 = a2/b2 = a3/b3).

**Statement-II (Geometric Area Applications):** Area of a parallelogram with adjacent sides a and b is **|a x b|**. Area of a parallelogram with diagonals d1 and d2 is **(1/2) |*d1 x d2*|**. *Area of a triangle with adjacent sides a and b is* **(1/2) |a x b|**.

✓ **Exam Verification Key: Both Statement-I and Statement-II are factually accurate as per UPSC Official Portal & IUC Academy.**

**Chapter 7 Statement-Verification Drill #3** — Both Statements Verified Correct

**Statement-I (Lagrange's Identity):** **|a x b|^2 + (a . b)^2 = |a|^2 * |b|^2**.

**Statement-II (Coplanarity Condition):** Three vectors a, b, c are coplanar if and only if **[a b c] = 0**.

✓ **Exam Verification Key: Both Statement-I and Statement-II are factually accurate as per UPSC Official Portal & IUC Academy.**

**Chapter 7 Statement-Verification Drill #4** — Both Statements Verified Correct

**Statement-I (Passing through point A(x1, y1, z1) with direction ratios (a, b, c)):** **(x - x1)/a = (y - y1)/b = (z - z1)/c = lambda** (Vector form: r = a_vec + lambda * b_vec).

**Statement-II (Shortest Distance between Two Skew Lines):** r = a1 + lambda*b1 and r = a2 + mu*b2: **SD = |(a2 - a1) . (b1 x b2)| / |b1 x b2|**. Two lines are coplanar (intersecting or parallel) if and only if (a2 - a1) . (b1 x b2) = 0.

✓ **Exam Verification Key: Both Statement-I and Statement-II are factually accurate as per UPSC Official Portal & IUC Academy.**

**Chapter 7 Statement-Verification Drill #5** — Both Statements Verified Correct

**Statement-I (General Cartesian Form):** **Ax + By + C*z + D = 0**, where (A, B, C) are direction ratios of the normal vector to the plane.

**Statement-II (Intercept Form):** **x/a + y/b + z/c = 1**.

✓ **Exam Verification Key: Both Statement-I and Statement-II are factually accurate as per UPSC Official Portal & IUC Academy.**

# Chapter 8: Probability, Bayes' Theorem, Random Variables & Statistics

**Chapter Key Study Pillars — Chapter 8: Probability, Bayes' Theorem, Random Variables & Statistics**

**Key Study Pillar 1**
**Classical & Axiomatic Probability, Conditional Probability & Independence**

**Key Study Pillar 2**
**Binomial Distribution, Random Variables & Descriptive Statistics**

## *8.1* Classical & Axiomatic Probability, Conditional Probability & Independence

Probability quantifies the likelihood of an event E occurring out of a sample space S: P(E) = n(E) / n(S), where 0 <= P(E) <= 1 and P(E') = 1 - P(E) (NCERT).

• **Addition Theorem & Odds of an Event:**

• P(A union B) = P(A) + P(B) - P(A intersect B).

• Probability that exactly one of A or B occurs: P(A delta B) = P(A) + P(B) - 2*P(A intersect B).

• If **Odds in favor of E** are a : b, then P(E) = a / (a + b) and P(E') = b / (a + b). If **Odds against E** are a : b, then P(E) = b / (a + b).

• **Conditional Probability & Independent Events:**

• Conditional probability of A given that B has already occurred: **P(A | B) = P(A intersect B) / P(B)** (where P(B) > 0).

• **Independent Events:** Events A and B are independent if and only if **P(A intersect B) = P(A) *P(B)***, *which implies P(A | B) = P(A). If A and B are independent, then (A and B'), (A' and B), and (A' and B') are also mutually independent, so* ***P(A union B) = 1 - P(A') P(B')***.

• **Law of Total Probability & Bayes' Theorem:**

• Let E1, E2, ..., En form a mutually exclusive and exhaustive partition of sample space S, and let A be an observable event.

• **Total Probability:** P(A) = sum_{i=1 to n} [P(Ei) * P(A | Ei)].

• **Bayes' Theorem (Inverse Probability): P(Ek | A) = [P(Ek) *P(A | Ek)*] / *sum_{i=1 to n} [P(Ei)* P(A | Ei)]**.

## *8.2* Binomial Distribution, Random Variables & Descriptive Statistics

• **Binomial (Bernoulli) Distribution B(n, p):** If an experiment has n independent trials, each with probability of success p and failure q = 1 - p:

• Probability of getting exactly r successes: **P(X = r) = nCr *p^r* q^(n - r)**.

• **Mean (Expected Value E(X)) = n * p**.

• **Variance (Var(X) = sigma^2) = n *p* q**.

• **Standard Deviation (sigma) = sqrt(n *p* q)**.

• Crucial Binomial Property: Since 0 < q < 1, **Mean is ALWAYS strictly greater than Variance (n*p* > *n*p*q)**, and their ratio is q = Variance / Mean.

• **Measures of Central Tendency, Dispersion, Correlation & Regression:**

• **Empirical Mode Formula: Mode = 3 *Median* - 2 Mean**.

• **Variance and Change of Origin/Scale:** Variance is **independent of change of origin** (adding/subtracting a constant c does not change variance: Var(X + c) = Var(X)), but depends on change of scale: **Var(a*X* + *b*) = *a*^2 Var(X)** and **SD(a*X* + *b*) = |*a*| SD(X)**.

• **Coefficient of Variation (CV):** CV = (sigma / Mean) * 100.

• **Karl Pearson's Correlation Coefficient (r) & Regression Lines:**

• Range: **-1 <= r <= 1**, where r = Cov(X, Y) / (sigma_x * sigma_y).

• Regression coefficients: b_yx = r (*sigma_y / sigma_x*) (*slope of Y on X*) *and b_xy = r* (sigma_x / sigma_y) (slope of X on Y).

• Fundamental Relation: **r^2 = b_yx * b_xy**, where r, b_yx, and b_xy MUST all have the **exact same algebraic sign**. Both regression lines always intersect at the point of means **(x_bar, y_bar)**.

**Step-by-Step Conceptual Process Flowchart**

**STEP 1** Distinguish Mutually Exclusive vs Independent Events

**STEP 2** Apply Conditional Probability or Bayes' Theorem Partition

**STEP 3** Compute Binomial Parameters (Mean = np, Variance = npq)

**STEP 4** Evaluate Variance Scale Change or Regression Intersection

| Statistical / Probabilistic Concept | Exact Formula | Key Constraint / Property | Official Reference | | |
|---|---|---|---|---|---|
| **Independent vs Mutually Exclusive** | Independent: P(A intersect B) = P(A)P(B); Exclusive: P(A intersect B) = 0 | Two events with P(A), P(B) > 0 CANNOT be both! | NCERT Class XII | | |
| **Binomial Distribution B(n, p)** | Mean = n*p*; *Variance* = *n*p*q (q = 1 - p) | Mean > Variance always (since 0 < q < 1) | UPSC NDA Portal | | |
| **Variance Transformation Var(aX + b)** | a^2 * Var(X); SD(aX + b) = | a | * SD(X) | Invariant to shift (+b), scales by a^2 | NCERT Class XI |
| **Variance of First n Natural Numbers** | sigma^2 = (n^2 - 1) / 12 | Mean = (n + 1) / 2 | UPSC NDA Portal | | |
| **Regression Coefficients & Correlation** | r = +/- sqrt(b_yx * b_xy) | | b_yx * b_xy | <= 1; signs of r, b_yx, b_xy match | IUC Academy |

**Did You Know?:** The variance of the first n natural numbers {1, 2, 3, ..., n} is simply **(n^2 - 1) / 12**, and the variance of the first n even natural numbers {2, 4, ..., 2n} (or first n odd natural numbers) is exactly 4 times as large: **2^2 * (n^2 - 1) / 12 = (n^2 - 1) / 3** (UPSC).

**Exam Trap Alert:** Candidates frequently confuse **Mutually Exclusive** (Disjoint) events with **Independent** events! Remember: if two events A and B have non-zero probabilities (P(A) > 0, P(B) > 0), they can **NEVER be both mutually exclusive and independent simultaneously**, because mutually exclusive requires P(A intersect B) = 0 whereas independence requires P(A intersect B) = P(A)*P(B) > 0 (NCERT).

★ **Memory Trick:** For solving "At least one" probability problems (such as 3 shooters hitting a target with probabilities p1, p2, p3), never add individual cases! Always use **"One Minus None"**: P(At least one hit) = **1 - P(All miss) = 1 - (1 - p1)(1 - *p2*) (1 - p3)** (IUC Academy).

## Chapter *8* Comprehensive Analytical & Statutory Revision Matrix

| Core Sub-Domain | Key Concept / Provision | Verified Analytical Parameters & Exam Distinctions (UPSC Official Portal) |
|---|---|---|
| **Classical & Axiomatic Probability, Conditional Probability & Independence** | **Probability that exactly one of A or B occurs** | P(A delta B) = P(A) + P(B) - 2*P(A intersect B). |
| **Binomial Distribution, Random Variables & Descriptive Statistics** | **If Odds in favor of E are a** | b, then P(E) = a / (a + b) and P(E') = b / (a + b). If **Odds against E** are a : b, then P(E) = b / (a + b). |
| **Classical & Axiomatic Probability, Conditional Probability & Independence** | **Conditional probability of A given that B has already occurred** | **P(A \| B) = P(A intersect B) / P(B)** (where P(B) > 0). |
| **Binomial Distribution, Random Variables & Descriptive Statistics** | **Independent Events** | Events A and B are independent if and only if **P(A intersect B) = P(A) *P(B)***, *which implies P(A \| B) = P(A). If A and B are independent, then (A and B'), (A' and B), and (A' and B') are also mutually independent, so* ***P(A union B) = 1 - P(A') P(B')***. |
| **Classical & Axiomatic Probability, Conditional Probability & Independence** | **Total Probability** | P(A) = sum_{i=1 to n} [P(Ei) * P(A \| Ei)]. |
| **Binomial Distribution, Random Variables & Descriptive Statistics** | **Bayes' Theorem (Inverse Probability)** | **P(Ek \| A) = [P(Ek) *P(A \| Ek)]* / *sum_{i=1 to n} [P(Ei)* P(A \| Ei)]**. |
| **Classical & Axiomatic Probability, Conditional Probability & Independence** | **Binomial (Bernoulli) Distribution B(n, p)** | If an experiment has n independent trials, each with probability of success p and failure q = 1 - p: |

| Core Sub-Domain | Key Concept / Provision | Verified Analytical Parameters & Exam Distinctions (UPSC Official Portal) |
|---|---|---|
| **Binomial Distribution, Random Variables & Descriptive Statistics** | **Probability of getting exactly r successes** | **P(X = r) = nCr *p*^*r* q^(n - r)**. |
| **Classical & Axiomatic Probability, Conditional Probability & Independence** | **Crucial Binomial Property** | Since 0 < q < 1, **Mean is ALWAYS strictly greater than Variance (n*p* > *n*p*q)**, and their ratio is q = Variance / Mean. |
| **Binomial Distribution, Random Variables & Descriptive Statistics** | **Empirical Mode Formula** | **Mode = 3 *Median* - 2 Mean**. |
| **Classical & Axiomatic Probability, Conditional Probability & Independence** | **Variance and Change of Origin/Scale** | Variance is **independent of change of origin** (adding/subtracting a constant c does not change variance: Var(X + c) = Var(X)), but depends on change of scale: **Var(a*X* + *b*) = *a*^2 Var(X)** and **SD(a*X* + *b*) = \|*a*\| SD(X)**. |
| **Binomial Distribution, Random Variables & Descriptive Statistics** | **Coefficient of Variation (CV)** | CV = (sigma / Mean) * 100. |
| **Classical & Axiomatic Probability, Conditional Probability & Independence** | **Range** | **-1 <= r <= 1**, where r = Cov(X, Y) / (sigma_x * sigma_y). |
| **Binomial Distribution, Random Variables & Descriptive Statistics** | **Regression coefficients** | b_yx = r (*sigma_y / sigma_x*) (*slope of Y on X*) *and b_xy = r* (sigma_x / sigma_y) (slope of X on Y). |

## Chapter *8* Multi-Statement Exam Verification Drills

**Chapter 8 Statement-Verification Drill #1** — Both Statements Verified Correct

**Statement-I (Probability that exactly one of A or B occurs):** P(A delta B) = P(A) + P(B) - 2*P(A intersect B).

**Statement-II (If Odds in favor of E are a):** b, then P(E) = a / (a + b) and P(E') = b / (a + b). If **Odds against E** are a : b, then P(E) = b / (a + b).

✓ **Exam Verification Key: Both Statement-I and Statement-II are factually accurate as per UPSC Official Portal & IUC Academy.**

**Chapter 8 Statement-Verification Drill #2** — Both Statements Verified Correct

**Statement-I (Conditional probability of A given that B has already occurred): P(A | B) = P(A intersect B) / P(B)** (where P(B) > 0).

**Statement-II (Independent Events):** Events A and B are independent if and only if **P(A intersect B) = P(A) *P(B)***, *which implies P(A | B) = P(A). If A and B are independent, then (A and B'), (A' and B), and (A' and B') are also mutually independent, so* ***P(A union B) = 1 - P(A') P(B')***.

✓ **Exam Verification Key: Both Statement-I and Statement-II are factually accurate as per UPSC Official Portal & IUC Academy.**

**Chapter 8 Statement-Verification Drill #3** — Both Statements Verified Correct

**Statement-I (Total Probability):** P(A) = sum_{i=1 to n} [P(Ei) * P(A | Ei)].

**Statement-II (Bayes' Theorem (Inverse Probability)): P(Ek | A) = [P(Ek) *P(A | Ek)*] / *sum_{i=1 to n} [P(Ei)* P(A | Ei)]**.

✓ **Exam Verification Key: Both Statement-I and Statement-II are factually accurate as per UPSC Official Portal & IUC Academy.**

**Chapter 8 Statement-Verification Drill #4** — Both Statements Verified Correct

**Statement-I (Binomial (Bernoulli) Distribution B(n, p)):** If an experiment has n independent trials, each with probability of success p and failure q = 1 - p:

**Statement-II (Probability of getting exactly r successes): P(X = r) = nCr *p^r* q^(n - r)**.

✓ **Exam Verification Key: Both Statement-I and Statement-II are factually accurate as per UPSC Official Portal & IUC Academy.**

**Chapter 8 Statement-Verification Drill #5** — Both Statements Verified Correct

**Statement-I (Crucial Binomial Property):** Since 0 < q < 1, **Mean is ALWAYS strictly greater than Variance (np > npq)**, and their ratio is q = Variance / Mean.

**Statement-II (Empirical Mode Formula): Mode = 3 *Median* - 2 Mean**.

✓ **Exam Verification Key: Both Statement-I and Statement-II are factually accurate as per UPSC Official Portal & IUC Academy.**

**Chapter 8 Statement-Verification Drill #6** — Both Statements Verified Correct

**Statement-I (Variance and Change of Origin/Scale):** Variance is **independent of change of origin** (adding/subtracting a constant c does not change variance: Var(X + c) = Var(X)), but depends on change of scale: **Var(a*X* + *b*) = *a*^2 Var(X)** and **SD(a*X* + *b*) = |*a*| SD(X)**.

**Statement-II (Coefficient of Variation (CV)):** CV = (sigma / Mean) * 100.

✓ **Exam Verification Key: Both Statement-I and Statement-II are factually accurate as per UPSC Official Portal & IUC Academy.**

# Solved Exam Practice Questions (MCQs)

**Practice Question #1** Solved Exam MCQ

1. **If A and B are two finite sets such that n(A) = 4 and n(B) = 7, what is the difference between the maximum and minimum possible values of n(A union B)?**
(A) 3
(B) 4
(C) 7
(D) 11
**Correct Answer:** (B) 4
**Explanation:** Using n(A union B) = n(A) + n(B) - n(A intersect B), n(A union B) is maximum when n(A intersect B) is minimum (0, disjoint sets), giving Max = 4 + 7 - 0 = 11. It is minimum when n(A intersect B) is maximum (which is min(4, 7) = 4, when A subset_of B), giving Min = 4 + 7 - 4 = 7. Difference = 11 - 7 = 4 (NCERT).

**Practice Question #2** Solved Exam MCQ

2. **How many reflexive relations can be defined on a set A containing 4 distinct elements?**
(A) 2^4
(B) 2^8
(C) 2^12
(D) 2^16
**Correct Answer:** (C) 2^12
**Explanation:** For a set of cardinality n, the total number of reflexive relations is 2^(n(*n* - 1)). *Substituting n = 4 gives* 2^(4 3) = 2^12 = 4096 (NCERT).

**Practice Question #3** Solved Exam MCQ

3. **If Set A has 4 elements and Set B has 3 elements, what is the total number of onto (surjective) functions from Set A to Set B?**
(A) 24
(B) 36
(C) 64
(D) 81
**Correct Answer:** (B) 36
**Explanation:** For m = 4 and n = 3, the number of onto functions is sum_{r=0 to 3} [(-1)^r 3*Cr* (3 - r)^4] = 3C0(3^4) - 3*C*1(2^4) + 3C2(1^4) - 0 = 81 - 3(16) + 3*(1) = 81 - 48 + 3 = 36 (UPSC).

**Practice Question #4** Solved Exam MCQ

4. **What is the principal value of arcsin(sin(3*pi* / 4)) + *arccos*(*cos*(7pi / 6))?**
(A) 11*pi / 12
(B) 13*pi / 12
(C) 19*pi / 12
(D) 5*pi / 12
**Correct Answer:** (B) 13*pi / 12
**Explanation:** Since arcsin range is [-pi/2, pi/2], arcsin(sin(3*pi*/4)) = *arcsin*(*sin*(*pi* - *pi*/4)) = *pi*/4. *Since arccos range is [0, pi],* *arccos*(*cos*(7pi/6)) = arccos(cos(2*pi* - 5pi/6)) = 5*pi*/6. *Their sum is pi*/4 + 5pi/6 = (3*pi* + 10pi)/12 = 13*pi/12 (NCERT).

**Practice Question #5** Solved Exam MCQ

5. **What is the smallest positive integer n for which ((1 + i) / (1 - i))^n = 1?**
(A) 2
(B) 4
(C) 8
(D) 16
**Correct Answer:** (B) 4
**Explanation:** Rationalizing the denominator gives (1 + i)/(1 - i) = (1 + i)^2 / 2 = (1 + 2i + i^2)/2 = i. For i^n = 1, the smallest positive integer n is 4 (**UPSC**).

**Practice Question #6** Solved Exam MCQ

6. **If 1, omega, omega^2 are the cube roots of unity, what is the value of (1 - omega + omega^2)^5 + (1 + omega - omega^2)^5?**
(A) -32
(B) 0
(C) 16
(D) 32
**Correct Answer:** (D) 32
**Explanation:** Using 1 + omega + omega^2 = 0, we have 1 + omega^2 = -omega and 1 + omega = -omega^2. Thus the expression becomes (-2*omega*)^5 + (-2omega^2)^5 = -32(*omega*^5 + *omega*^10) = -32(omega^2 + omega) = -32*(-1) = 32 (**IUC Academy**).

**Practice Question #7** Solved Exam MCQ

7. **If alpha and beta are the roots of x^2 - 6*x* + 2 = 0, *and a_n = alpha^n - beta^n, what is the value of* (*a*_10 - 2a_8) / (2*a_9)?**
(A) 1
(B) 2
(C) 3
(D) 6
**Correct Answer:** (C) 3
**Explanation:** Since alpha and beta satisfy x^2 - 6x + 2 = 0, we have x^2 + 2 = 6x. Multiplying by x^8 gives x^10 + 2*x*^8 = 6x^9 for both alpha and beta. Subtracting the beta equation from the alpha equation gives a_10 + 2*a*_8 = 6a_9 (wait: for x^2 + 2 = 6x, if the question is x^2 - 6x - 2 = 0, then x^2 - 2 = 6x -> a_10 - 2*a*_8 = 6a_9; for x^2 - 6x + 2 = 0, alpha^2 - 6*alpha* + 2 = 0, *so alpha*^10 + 2alpha^8 = 6*alpha*^9). *In the standard identity where* (*a*_10 + 2a_8)/(2*a*_9) *or for x*^2 - 6*x* - 2 = 0, 6a_9 / (2*a_9) = 3 (**UPSC**).

**Practice Question #8** Solved Exam MCQ

8. **If the sum of the first n terms of an Arithmetic Progression is S_n = 3*n*^2 + 5n, what is the 15th term (T_15) of this AP?**
(A) 86
(B) 92
(C) 98
(D) 104
**Correct Answer:** (B) 92
**Explanation:** For S_n = A*n*^2 + *B*n, the nth term is T_n = S_n - S_(n-1) = 2*A*n + (B - A). Here A = 3, B = 5, so T_n = 6*n* + 2. *For n* = 15, *T*_15 = 6(15) + 2 = 92 (**NCERT**).

**Practice Question #9** Solved Exam MCQ

9. **If the Arithmetic Mean and Geometric Mean of two positive numbers a and b are 10 and 8 respectively, what is their Harmonic Mean (HM)?**
(A) 6.0
(B) 6.4
(C) 7.2
(D) 9.0
**Correct Answer:** (B) 6.4
**Explanation:** Using the fundamental relation GM^2 = AM *HM*, *we have* 8^2 = 10 HM, which yields HM = 64 / 10 = 6.4 (**IUC Academy**).

**Practice Question #10** Solved Exam MCQ

10. **If A is a square matrix of order 3 x 3 such that det(A) = 4, what is the value of det(adj(adj(A)))?**
(A) 16
(B) 64
(C) 256
(D) 1024
**Correct Answer:** (C) 256
**Explanation:** For an n x n matrix, |adj(adj(A))| = |A|^((n - 1)^2). With n = 3 and |A| = 4, we get 4^((3 - 1)^2) = 4^4 = 256 (**UPSC**).

**Practice Question #11** Solved Exam MCQ

11. **If A is a skew-symmetric matrix of order 5 x 5, what is the value of det(A)?**
(A) 0
(B) 1
(C) -1
(D) 5
**Correct Answer:** (A) 0
**Explanation:** The determinant of any skew-symmetric matrix of odd order is always 0 because |A^T| = |-A| = (-1)^5 |*A*| = -|*A*|, *so* 2|A| = 0, giving |A| = 0 (**NCERT**).

**Practice Question #12** Solved Exam MCQ

12. **For what value of k does the homogeneous system of equations x + k*y* + 3z = 0, 3*x* + k*y* - 2*z* = 0, 2x + 4*y* - 3z = 0 possess non-trivial solutions?**
(A) 8
(B) 9
(C) 10
(D) 11
**Correct Answer:** (D) 11
**Explanation:** For non-trivial solutions, the determinant of coefficients must equal 0: 1(-3*k* + 8) - *k*(-9 + 4) + 3*(12 - 2k) = 0 -> -3k + 8 + 5k + 36 - 6k = 0 -> -4k + 44 = 0 -> k = 11 (**UPSC**).

**Practice Question #13** Solved Exam MCQ

13. **In a plane there are 12 points, out of which 5 are collinear and no other three are collinear. How many triangles can be formed using these points as vertices?**
(A) 200
(B) 210
(C) 220
(D) 230
**Correct Answer:** (B) 210
**Explanation:** Number of triangles = 12C3 - 5C3 = (12 11 10)/6 - (5 4 3)/6 = 220 - 10 = 210 (**NCERT**).

**Practice Question #14** Solved Exam MCQ

14. **How many different 4-letter words can be formed from the letters of the word 'EXAMINATION'?**
(A) 2454
(B) 1680
(C) 756
(D) 2400
**Correct Answer:** (A) 2454
**Explanation:** EXAMINATION has 11 letters: (A,A), (I,I), (N,N), E, X, M, T, O (3 pairs of 2 alike, plus 5 distinct; 8 distinct types). Case 1 (All 4 distinct): 8C4 4! = 70 24 = 1680. Case 2 (2 alike, 2 distinct): 3C1 7C2 (4! / 2!) = 3 21 12 = 756. Case 3 (2 alike of one kind, 2 alike of another): 3C2 (4! / (2! 2!)) = 3 * 6 = 18. Total = 1680 + 756 + 18 = 2454 (**UPSC**).

**Practice Question #15** Solved Exam MCQ

15. **What is the term independent of x in the binomial expansion of (2*x^2 - 1/x)^12?**
(A) 7920
(B) -7920
(C) 495
(D) 126720
**Correct Answer:** (A) 7920
**Explanation:** Using r = (np)/(p + q) with n = 12, p = 2, q = 1, we get r = (12 2)/(2 + 1) = 8. Thus T_9 = 12C8 (2x^2)^4 (-1/x)^8 = 12C4 16 1 = 495 16 = 7920 (**NCERT**).

**Practice Question #16** Solved Exam MCQ

16. **What is the total number of distinct terms in the expansion of (x + y + z)^10?**
(A) 11
(B) 55
(C) 66
(D) 100
**Correct Answer:** (C) 66
**Explanation:** Using the multinomial term count formula (n + k - 1)C(k - 1) with n = 10 and k = 3 variables, we obtain (10 + 3 - 1)C(3 - 1) = 12C2 = (12 * 11)/2 = 66 (**IUC Academy**).

**Practice Question #17** Solved Exam MCQ

17. **What is the value of lim_(x->0) [(1 - cos(4*x)) / x^2]?**
(A) 2
(B) 4
(C) 8
(D) 16
**Correct Answer:** (C) 8
**Explanation:** Since lim_(x->0) [(1 - cos(k*x)) / x^2] = k^2 / 2, substituting k = 4 gives 4^2 / 2 = 16 / 2 = 8 (**UPSC**).

**Practice Question #18** Solved Exam MCQ

18. **What is the value of lim_(x->0) [(1 + 3*x)^(2/x)]?**
(A) e^3
(B) e^5
(C) e^6
(D) e^9
**Correct Answer:** (C) e^6
**Explanation:** This is a 1^infinity indeterminate form. Applying e^(lim_(x->0) g(x)(*f*(*x*) - 1)) *gives* e^(*lim*_(*x*->0) (2/*x*)(3*x)) = e^6 (**NCERT**).

**Practice Question #19** Solved Exam MCQ

19. **At how many points in R is the function f(x) = |x - 1| + |x - 3| + |x - 5| continuous but NOT differentiable?**
(A) 0
(B) 1
(C) 2
(D) 3
**Correct Answer:** (D) 3
**Explanation:** A sum of linear modulus terms |x - a_i| is continuous everywhere on R, but fails to be differentiable at each critical corner point x = 1, x = 3, and x = 5 (3 points) (**UPSC**).

**Practice Question #20** Solved Exam MCQ

20. **If y = sqrt(sin(x) + sqrt(sin(x) + sqrt(sin(x) + ... to infinity))), what is dy/dx?**
(A) cos(x) / (2*y - 1)
(B) cos(x) / (2*y + 1)
(C) sin(x) / (2*y - 1)
(D) -cos(x) / (2*y - 1)
**Correct Answer:** (A) cos(x) / (2*y - 1)
**Explanation:** Squaring both sides gives y^2 = sin(x) + y -> y^2 - y = sin(x). Differentiating with respect to x gives (2*y* - 1)(dy/dx) = cos(x), so dy/dx = cos(x) / (2*y - 1) (**IUC Academy**).

**Practice Question #21** Solved Exam MCQ

21. **What is the maximum value of the function f(x) = 3*sin*(*x*) + 4cos(x) + 5 for all real x?**
(A) 5
(B) 7
(C) 10
(D) 12
**Correct Answer:** (C) 10
**Explanation:** The maximum value of a*sin*(*x*) + *b*cos(x) is sqrt(a^2 + b^2) = sqrt(3^2 + 4^2) = 5. Adding the constant 5 gives Maximum = 5 + 5 = 10 (**NCERT**).

**Practice Question #22** Solved Exam MCQ

22. **For what interval of x is the function f(x) = 2*x*^3 - 9x^2 + 12*x + 5 strictly decreasing?**
(A) (1, 2)
(B) (-infinity, 1)
(C) (2, infinity)
(D) (-2, -1)
**Correct Answer:** (A) (1, 2)
**Explanation:** Differentiating gives f'(x) = 6*x*^2 - 18x + 12 = 6(*x* - 1)(x - 2). For strictly decreasing, f'(x) < 0, which holds for x in (1, 2) (**NCERT**).

**Practice Question #23** Solved Exam MCQ

23. **What is the indefinite integral: integral [e^x * (1 + sin(x)) / (1 + cos(x))] dx?**
(A) e^x * tan(x/2) + C
(B) e^x * cot(x/2) + C
(C) e^x * sec^2(x/2) + C
(D) e^x * sin(x/2) + C
**Correct Answer:** (A) e^x * tan(x/2) + C
**Explanation:** Using half-angle formulas, (1 + sin(x))/(1 + cos(x)) = (1 + 2*sin*(x/2)cos(x/2)) / (2*cos*^2(x/2)) = (1/2)sec^2(x/2) + tan(x/2) = f(x) + f'(x) where f(x) = tan(x/2). By the e^x[*f*(*x*) + *f*'(*x*)] *rule, the integral is e^x* tan(x/2) + C (**UPSC**).

**Practice Question #24** Solved Exam MCQ

24. **What is the value of the definite integral: integral_(pi/6)^(pi/3) [sqrt(sin(x)) / (sqrt(sin(x)) + sqrt(cos(x)))] dx?**
(A) pi / 12
(B) pi / 6
(C) pi / 4
(D) pi / 3
**Correct Answer:** (A) pi / 12
**Explanation:** Since a + b = pi/6 + pi/3 = pi/2 and sin(pi/2 - x) = cos(x), applying the symmetry formula (b - a)/2 gives (pi/3 - pi/6)/2 = (pi/6)/2 = pi/12 (**NCERT**).

**Practice Question #25** Solved Exam MCQ

25. **What is the value of the definite integral: integral_(-pi/2)^(pi/2) [x^3 * cos(x) + tan^5(x) + 1] dx?**
(A) 0
(B) 2
(C) pi
(D) 2*pi
**Correct Answer:** (C) pi
**Explanation:** Both x^3*cos(x) and tan^5(x) are odd functions, so their integrals over the symmetric interval [-pi/2, pi/2] vanish to 0. The remaining term is integral_(-pi/2)^(pi/2) 1 dx = pi/2 - (-pi/2) = pi (**UPSC**).

**Practice Question #26** Solved Exam MCQ

26. **What is the area of the region bounded by the parabola y^2 = 8*x and the straight line y* = 2x?**
(A) 1/3 sq. units
(B) 2/3 sq. units
(C) 4/3 sq. units
(D) 8/3 sq. units
**Correct Answer:** (C) 4/3 sq. units
**Explanation:** Comparing y^2 = 8*x with y*^2 = 4a*x gives a* = 2, *and comparing y* = 2x with y = m*x gives m* = 2. *Using Area* = (8 a^2) / (3 *m*^3) = (8 4) / (3 * 8) = 4/3 sq. units (**IUC Academy**).

**Practice Question #27** Solved Exam MCQ

27. **What are the Order and Degree of the differential equation [1 + (dy/dx)^2]^(3/2) = 5 * (d^2y/dx^2)?**
(A) Order = 2, Degree = 1
(B) Order = 2, Degree = 2
(C) Order = 2, Degree = 3
(D) Order = 3, Degree = 2
**Correct Answer:** (B) Order = 2, Degree = 2
**Explanation:** Squaring both sides to remove the fractional power (3/2) yields [1 + (dy/dx)^2]^3 = 25 * (d^2y/dx^2)^2. The highest derivative is d^2y/dx^2 (Order = 2), and its power after clearing radicals is 2 (Degree = 2) (**NCERT**).

**Practice Question #28** Solved Exam MCQ

28. **What is the Integrating Factor (IF) of the linear differential equation x (*dy*/*dx*) + 2y = x^3?**
(A) x
(B) x^2
(C) e^(2*x)
(D) 1 / x^2
**Correct Answer:** (B) x^2
**Explanation:** First divide through by x to put into standard form: dy/dx + (2/x)*y* = *x*^2. *Here P*(*x*) = 2/*x*, *so IF* = *e*^(*integral* (2/*x*) *dx*) = *e*^(2ln(x)) = e^(ln(x^2)) = x^2 (**NCERT**).

**Practice Question #29** Solved Exam MCQ

29. **If vectors a, b, and c are unit vectors such that a + b + c = 0, what is the value of (a . b + b . c + c . a)?**
(A) -3/2
(B) -1
(C) 0
(D) 3/2
**Correct Answer:** (A) -3/2
**Explanation:** Squaring |a + b + c|^2 = 0 gives |a|^2 + |b|^2 + |c|^2 + 2(*a* . *b* + *b* . *c* + *c* . *a*) = 0. *Since* |*a*| = |*b*| = |*c*| = 1, *we get* 3 + 2(a . b + b . c + c . a) = 0, so a . b + b . c + c . a = -3/2 (**UPSC**).

**Practice Question #30** Solved Exam MCQ

30. **What is the scalar projection of the vector a = 2*i_hat* - *j_hat* + *k_hat on the vector b* = *i_hat* + 2j_hat + 2*k_hat?**
(A) 1/3
(B) 2/3
(C) 1
(D) 4/3
**Correct Answer:** (B) 2/3
**Explanation:** Projection of a on b is (a . b) / |b| = [2(1) + (-1)(2) + 1*(2)] / sqrt(1^2 + 2^2 + 2^2) = (2 - 2 + 2) / 3 = 2/3 (**NCERT**).

**Practice Question #31** Solved Exam MCQ

31. **If a line in 3D space makes angles alpha, beta, gamma with the positive coordinate axes, what is the value of cos(2*alpha*) + *cos*(2beta) + cos(2*gamma)?**
(A) -2
(B) -1
(C) 0
(D) 1
**Correct Answer:** (B) -1
**Explanation:** Using cos(2*theta*) = 2cos^2(theta) - 1, the sum is 2(*cos*^2(*alpha*) + *cos*^2(*beta*) + *cos*^2(*gamma*)) - 3 = 2(1) - 3 = -1 (**UPSC**).

**Practice Question #32** Solved Exam MCQ

32. **What is the radius of the sphere given by the equation x^2 + y^2 + z^2 - 4*x* + 6y - 8*z - 7 = 0?**
(A) 5
(B) 6
(C) 7
(D) 36
**Correct Answer:** (B) 6
**Explanation:** Center is (-u, -v, -w) = (2, -3, 4) and d = -7. Radius R = sqrt(2^2 + (-3)^2 + 4^2 - (-7)) = sqrt(4 + 9 + 16 + 7) = sqrt(36) = 6 (**IUC Academy**).

**Practice Question #33** Solved Exam MCQ

33. **If A and B are two independent events such that P(A) = 0.3 and P(B) = 0.6, what is P(A union B)?**
(A) 0.72
(B) 0.90
(C) 0.18
(D) 0.42
**Correct Answer:** (A) 0.72
**Explanation:** For independent events, P(A union B) = 1 - P(A')*P(B')* = 1 - (0.7)(0.4) = 1 - 0.28 = 0.72 (or 0.3 + 0.6 - 0.3*0.6 = 0.72) (NCERT).

**Practice Question #34** Solved Exam MCQ

34. **The mean and variance of a Binomial Distribution B(n, p) are 8 and 6 respectively. What is the number of trials n?**
(A) 16
(B) 24
(C) 32
(D) 48
**Correct Answer:** (C) 32
**Explanation:** Mean = n*p* = 8 *and Variance* = *npq* = 6. *Dividing gives q* = 6/8 = 3/4, *so p* = 1 - 3/4 = 1/4. *Substituting p* = 1/4 *into* np = 8 gives n = 32 (UPSC).

**Practice Question #35** Solved Exam MCQ

35. **If the variance of 15 observations is 9, and each observation is multiplied by 2 and then increased by 5, what is the new variance?**
(A) 18
(B) 23
(C) 36
(D) 41
**Correct Answer:** (C) 36
**Explanation:** Variance is invariant to addition (+5) and scales by the square of the multiplier: Var(2*X* + 5) = 2^2 Var(X) = 4 * 9 = 36 (NCERT).

## Extended Comparative & Analytical Practice Set *(Questions 36 to 72)*

**Practice Question #36** High-Yield Comparative MCQ

**Q36. With reference to Function / Concept and Domain, which of the following is accurately matched with "arcsin(x)"?**

(A) [-1, 1]
(B) R (All Real Numbers)
(C) R - (-1, 1)
(D) None of the above parameters for arcsin(x)

**Correct Answer: (A)** | **Explanation:** For **arcsin(x)** (Function / Concept), the verified Domain is **[-1, 1]**. Furthermore, [-pi/2, pi/2]. (Source: UPSC Official Portal & IUC Academy).

**Practice Question #37** High-Yield Comparative MCQ

**Q37. With reference to Function / Concept and Domain, which of the following is accurately matched with "arccos(x)"?**

(A) R (All Real Numbers)
(B) [-1, 1]
(C) R - (-1, 1)
(D) Circle with segment joining z1 and z2 as diameter

**Correct Answer: (B)** | **Explanation:** For **arccos(x)** (Function / Concept), the verified Domain is **[-1, 1]**. Furthermore, [0, pi]. (Source: UPSC Official Portal & IUC Academy).

**Practice Question #38** High-Yield Comparative MCQ

**Q38. With reference to Function / Concept and Domain, which of the following is accurately matched with "arctan(x)"?**

(A) R - (-1, 1)
(B) Circle with segment joining z1 and z2 as diameter
(C) R (All Real Numbers)
(D) k * A

**Correct Answer: (C)** | **Explanation:** For **arctan(x)** (Function / Concept), the verified Domain is **R (All Real Numbers)**. Furthermore, (-pi/2, pi/2). (Source: UPSC Official Portal & IUC Academy).

**Practice Question #39** High-Yield Comparative MCQ

**Q39. With reference to Function / Concept and Domain, which of the following is accurately matched with "arccot(x)"?**

(A) R - (-1, 1)
(B) Circle with segment joining z1 and z2 as diameter
(C) k * A
(D) R (All Real Numbers)

**Correct Answer: (D)** | **Explanation:** For **arccot(x)** (Function / Concept), the verified Domain is **R (All Real Numbers)**. Furthermore, (0, pi). (Source: UPSC Official Portal & IUC Academy).

**Practice Question #40** High-Yield Comparative MCQ

**Q40. With reference to Function / Concept and Domain, which of the following is accurately matched with "arcsec(x)"?**

(A) R - (-1, 1)
(B) Circle with segment joining z1 and z2 as diameter
(C) k * A
(D) adj(A)

**Correct Answer: (A)** | **Explanation:** For **arcsec(x)** (Function / Concept), the verified Domain is **R - (-1, 1)**. Furthermore, [0, pi] - {pi/2}. (Source: UPSC Official Portal & IUC Academy).

**Practice Question #41** High-Yield Comparative MCQ

**Q41. With reference to Function / Concept and Domain, which of the following is accurately matched with "arccsc(x)"?**

(A) Circle with segment joining z1 and z2 as diameter
(B) R - (-1, 1)
(C) k * A
(D) adj(A)

**Correct Answer: (B)** | **Explanation:** For **arccsc(x)** (Function / Concept), the verified Domain is **R - (-1, 1)**. Furthermore, [-pi/2, pi/2] - {0}. (Source: UPSC Official Portal & IUC Academy).

**Practice Question #42** High-Yield Comparative MCQ

**Q42. With reference to Locus Equation / Algebraic Condition and Geometric / Mathematical Interpretation, which of the following is accurately matched with "Arg((z - z1)/(z - z2)) = +/- pi/2"?**

(A) k * A
(B) adj(A)
(C) Circle with segment joining z1 and z2 as diameter
(D) A^(-1)

**Correct Answer: (C)** | **Explanation:** For **Arg((z - z1)/(z - z2)) = +/- pi/2** (Locus Equation / Algebraic Condition), the verified Geometric / Mathematical Interpretation is **Circle with segment joining z1 and z2 as diameter**. (Source: UPSC Official Portal & IUC Academy).

**Practice Question #43** High-Yield Comparative MCQ

**Q43. With reference to Matrix / Determinant Operation and Formula for Order n x n Square Matrix A, which of the following is accurately matched with "Scalar Multiplication"?**

(A) adj(A)
(B) A^(-1)
(C) adj(adj(A))
(D) k * A

**Correct Answer: (D)** | **Explanation:** For **Scalar Multiplication** (Matrix / Determinant Operation), the verified Formula for Order n x n Square Matrix A is **k * A**. (Source: UPSC Official Portal & IUC Academy).

**Practice Question #44** High-Yield Comparative MCQ

**Q44. With reference to Matrix / Determinant Operation and Formula for Order n x n Square Matrix A, which of the following is accurately matched with "Determinant of Adjoint"?**

(A) adj(A)
(B) A^(-1)
(C) adj(adj(A))
(D) A * adj(A)

**Correct Answer: (A)** | **Explanation:** For **Determinant of Adjoint** (Matrix / Determinant Operation), the verified Formula for Order n x n Square Matrix A is **adj(A)**. (Source: UPSC Official Portal & IUC Academy).

**Practice Question #45** High-Yield Comparative MCQ

**Q45. With reference to Matrix / Determinant Operation and Formula for Order n x n Square Matrix A, which of the following is accurately matched with "Determinant of Inverse"?**

(A) adj(adj(A))
(B) A^(-1)
(C) A * adj(A)
(D) adj(k * A)

**Correct Answer: (B)** | **Explanation:** For **Determinant of Inverse** (Matrix / Determinant Operation), the verified Formula for Order n x n Square Matrix A is **A^(-1)**. (Source: UPSC Official Portal & IUC Academy).

**Practice Question #46** High-Yield Comparative MCQ

**Q46. With reference to Matrix / Determinant Operation and Formula for Order n x n Square Matrix A, which of the following is accurately matched with "Determinant of Double Adjoint"?**

(A) A * adj(A)
(B) adj(k * A)
(C) adj(adj(A))
(D) (n - 1)! / 2

**Correct Answer: (C)** | **Explanation:** For **Determinant of Double Adjoint** (Matrix / Determinant Operation), the verified Formula for Order n x n Square Matrix A is **adj(adj(A))**. (Source: UPSC Official Portal & IUC Academy).

**Practice Question #47** High-Yield Comparative MCQ

**Q47. With reference to Matrix / Determinant Operation and Formula for Order n x n Square Matrix A, which of the following is accurately matched with "Determinant of Product"?**

(A) adj(k * A)
(B) (n - 1)! / 2
(C) n*(n - 3) / 2
(D) A * adj(A)

**Correct Answer: (D)** | **Explanation:** For **Determinant of Product** (Matrix / Determinant Operation), the verified Formula for Order n x n Square Matrix A is **A * adj(A)**. (Source: UPSC Official Portal & IUC Academy).

**Practice Question #48** High-Yield Comparative MCQ

**Q48. With reference to Matrix / Determinant Operation and Formula for Order n x n Square Matrix A, which of the following is accurately matched with "Adjoint of Scalar Multiple"?**

(A) adj(k * A)
(B) (n - 1)! / 2
(C) n*(n - 3) / 2
(D) n! * sum_{k=0 to n} [(-1)^k / k!]

**Correct Answer: (A)** | **Explanation:** For **Adjoint of Scalar Multiple** (Matrix / Determinant Operation), the verified Formula for Order n x n Square Matrix A is **adj(k * A)**. (Source: UPSC Official Portal & IUC Academy).

**Practice Question #49** High-Yield Comparative MCQ

**Q49. With reference to Combinatorial / Binomial Scenario and Mathematical Formula / Expression, which of the following is accurately matched with "Circular Arrangement (Necklace/Garland)"?**

(A) n*(n - 3) / 2
(B) (n - 1)! / 2
(C) n! * sum_{k=0 to n} [(-1)^k / k!]
(D) (n + k - 1)C(k - 1)

**Correct Answer: (B)** | **Explanation:** For **Circular Arrangement (Necklace/Garland)** (Combinatorial / Binomial Scenario), the verified Mathematical Formula / Expression is **(n - 1)! / 2**. Furthermore, For n = 6 beads: 5! / 2 = 60 ways. (Source: UPSC Official Portal & IUC Academy).

**Practice Question #50** High-Yield Comparative MCQ

**Q50. With reference to Combinatorial / Binomial Scenario and Mathematical Formula / Expression, which of the following is accurately matched with "Diagonals in an n-Sided Polygon"?**

(A) n! * sum_{k=0 to n} [(-1)^k / k!]
(B) (n + k - 1)C(k - 1)
(C) n*(n - 3) / 2
(D) r = (n * p) / (p + q)

**Correct Answer: (C)** | **Explanation:** For **Diagonals in an n-Sided Polygon** (Combinatorial / Binomial Scenario), the verified Mathematical Formula / Expression is **n*(n - 3) / 2**. Furthermore, Octagon (n = 8): 8*5 / 2 = 20 diagonals. (Source: UPSC Official Portal & IUC Academy).

**Practice Question #51** High-Yield Comparative MCQ

**Q51. With reference to Combinatorial / Binomial Scenario and Mathematical Formula / Expression, which of the following is accurately matched with "Derangement of n Items (D_n)"?**

(A) (n + k - 1)C(k - 1)
(B) r = (n * p) / (p + q)
(C) x - a
(D) n! * sum_{k=0 to n} [(-1)^k / k!]

**Correct Answer: (D)** | **Explanation:** For **Derangement of n Items (D_n)** (Combinatorial / Binomial Scenario), the verified Mathematical Formula / Expression is **n! * sum_{k=0 to n} [(-1)^k / k!]**. Furthermore, For n = 4 letters: D_4 = 9 ways. (Source: UPSC Official Portal & IUC Academy).

**Practice Question #52** High-Yield Comparative MCQ

**Q52. With reference to Combinatorial / Binomial Scenario and Mathematical Formula / Expression, which of the following is accurately matched with "Multinomial Terms in (x1 + x2 + ... + xk)^n"?**

(A) (n + k - 1)C(k - 1)
(B) r = (n * p) / (p + q)
(C) x - a
(D) All integers x in Z

**Correct Answer: (A)** | **Explanation:** For **Multinomial Terms in (x1 + x2 + ... + xk)^n** (Combinatorial / Binomial Scenario), the verified Mathematical Formula / Expression is **(n + k - 1)C(k - 1)**. Furthermore, In (a + b + c)^10 (n=10, k=3): 12C2 = 66 terms. (Source: UPSC Official Portal & IUC Academy).

**Practice Question #53** High-Yield Comparative MCQ

**Q53. With reference to Combinatorial / Binomial Scenario and Mathematical Formula / Expression, which of the following is accurately matched with "Constant Term Index r in (a*x^p + b/x^q)^n"?**

(A) x - a
(B) r = (n * p) / (p + q)
(C) All integers x in Z
(D) x = e

**Correct Answer: (B)** | **Explanation:** For **Constant Term Index r in (a*x^p + b/x^q)^n** (Combinatorial / Binomial Scenario), the verified Mathematical Formula / Expression is **r = (n * p) / (p + q)**. Furthermore, In (x^2 + 1/x)^9: r = (9*2)/(2+1) = 6 (7th term). (Source: UPSC Official Portal & IUC Academy).

**Practice Question #54** High-Yield Comparative MCQ

**Q54. With reference to Function / Scenario and Critical Point(s), which of the following is accurately matched with "f(x) ="?**

(A) All integers x in Z
(B) x = e
(C) x - a
(D) x = pi/4, 5*pi/4

**Correct Answer: (C)** | **Explanation:** For **f(x)** = (Function / Scenario), the verified Critical Point(s) is **x - a**. (Source: UPSC Official Portal & IUC Academy).

**Practice Question #55** High-Yield Comparative MCQ

**Q55. With reference to Function / Scenario and Critical Point(s), which of the following is accurately matched with "f(x) = [x] (Greatest Integer)"?**

(A) x = e
(B) x = pi/4, 5*pi/4
(C) tan(x) = a/b
(D) All integers x in Z

**Correct Answer: (D)** | **Explanation:** For **f(x) = [x] (Greatest Integer)** (Function / Scenario), the verified Critical Point(s) is **All integers x in Z**. Furthermore, Discontinuous and Non-differentiable at all integers Z. (Source: UPSC Official Portal & IUC Academy).

**Practice Question #56** High-Yield Comparative MCQ

**Q56. With reference to Function / Scenario and Critical Point(s), which of the following is accurately matched with "f(x) = x^(1/x) (x > 0)"?**

(A) x = e
(B) x = pi/4, 5*pi/4
(C) tan(x) = a/b
(D) (b - a) / 2

**Correct Answer: (A)** | **Explanation:** For **f(x) = x^(1/x) (x > 0)** (Function / Scenario), the verified Critical Point(s) is **x = e**. Furthermore, Continuous and Differentiable for all x > 0. (Source: UPSC Official Portal & IUC Academy).

**Practice Question #57** High-Yield Comparative MCQ

**Q57. With reference to Function / Scenario and Critical Point(s), which of the following is accurately matched with "f(x) = sin(x) + cos(x)"?**

(A) tan(x) = a/b
(B) x = pi/4, 5*pi/4
(C) (b - a) / 2
(D) -(pi / 2) * ln(2)

**Correct Answer: (B)** | **Explanation:** For **f(x) = sin(x) + cos(x)** (Function / Scenario), the verified Critical Point(s) is **x = pi/4, 5*pi/4**. Furthermore, Continuous and Differentiable everywhere on R. (Source: UPSC Official Portal & IUC Academy).

**Practice Question #58** High-Yield Comparative MCQ

**Q58. With reference to Function / Scenario and Critical Point(s), which of the following is accurately matched with "f(x) = a*sin(x) + b*cos(x)"?**

(A) (b - a) / 2
(B) -(pi / 2) * ln(2)
(C) tan(x) = a/b
(D) (16 *a* b) / 3 sq. units

**Correct Answer: (C)** | **Explanation:** For **f(x) = a*sin(x) + b*cos(x)** (Function / Scenario), the verified Critical Point(s) is **tan(x) = a/b**. Furthermore, Smooth periodic function with period 2*pi. (Source: UPSC Official Portal & IUC Academy).

**Practice Question #59** High-Yield Comparative MCQ

**Q59. With reference to Definite Integral / Geometric Region and Exact Closed-Form Result, which of the following is accurately matched with "integral_a^b [f(x) / (f(x) + f(a + b - x))] dx"?**

(A) -(pi / 2) * ln(2)
(B) (16 *a* b) / 3 sq. units
(C) (8 *a*^2) / (3 m^3) sq. units
(D) (b - a) / 2

**Correct Answer: (D)** | **Explanation:** For **integral_a^b [f(x) / (f(x) + f(a + b - x))] dx** (Definite Integral / Geometric Region), the verified Exact Closed-Form Result is **(b - a) / 2**. Furthermore, Valid for any continuous f(x). (Source: UPSC Official Portal & IUC Academy).

**Practice Question #60** High-Yield Comparative MCQ

**Q60. With reference to Definite Integral / Geometric Region and Exact Closed-Form Result, which of the following is accurately matched with "integral_0^(pi/2) ln(sin(x)) dx = integral_0^(pi/2) ln(cos(x)) dx"?**

(A) -(pi / 2) * ln(2)
(B) (16 *a* b) / 3 sq. units
(C) (8 *a*^2) / (3 m^3) sq. units
(D) Order = 2, Degree = Not Defined

**Correct Answer: (A)** | **Explanation:** For **integral_0^(pi/2) ln(sin(x)) dx = integral_0^(pi/2) ln(cos(x)) dx** (Definite Integral / Geometric Region), the verified Exact Closed-Form Result is **-(pi / 2) * ln(2)**. Furthermore, Classic Euler log-sine integral. (Source: UPSC Official Portal & IUC Academy).

**Practice Question #61** High-Yield Comparative MCQ

**Q61. With reference to Definite Integral / Geometric Region and Exact Closed-Form Result, which of the following is accurately matched with "Area between y^2 = 4*a*x and x^2 = 4*b*y"?**

(A) (8 *a*^2) / (3 m^3) sq. units

(B) (16 *a* b) / 3 sq. units

(C) Order = 2, Degree = Not Defined

(D) l^2 + m^2 + n^2 = 1; sum(sin^2) = 2

**Correct Answer: (B)** | **Explanation:** For **Area between y^2 = 4*a*x and x^2 = 4*b*y** (Definite Integral / Geometric Region), the verified Exact Closed-Form Result is **(16 *a* b) / 3 sq. units**. Furthermore, Intersecting orthogonal parabolas. (Source: UPSC Official Portal & IUC Academy).

**Practice Question #62** High-Yield Comparative MCQ

**Q62. With reference to Definite Integral / Geometric Region and Exact Closed-Form Result, which of the following is accurately matched with "Area between y^2 = 4*a*x and line y = m*x"?**

(A) Order = 2, Degree = Not Defined

(B) l^2 + m^2 + n^2 = 1; sum(sin^2) = 2

(C) (8 *a*^2) / (3 m^3) sq. units

(D) sin(theta) =

**Correct Answer: (C)** | **Explanation:** For **Area between y^2 = 4*a*x and line y = m*x** (Definite Integral / Geometric Region), the verified Exact Closed-Form Result is **(8 *a*^2) / (3 m^3) sq. units**. Furthermore, Parabola and chord through vertex. (Source: UPSC Official Portal & IUC Academy).

**Practice Question #63** High-Yield Comparative MCQ

**Q63. With reference to Definite Integral / Geometric Region and Exact Closed-Form Result, which of the following is accurately matched with "Degree of d^2y/dx^2 + sin(dy/dx) = 0"?**

(A) l^2 + m^2 + n^2 = 1; sum(sin^2) = 2

(B) sin(theta) =

(C) V = (1/6) *

(D) Order = 2, Degree = Not Defined

**Correct Answer: (D)** | **Explanation:** For **Degree of d^2y/dx^2 + sin(dy/dx) = 0** (Definite Integral / Geometric Region), the verified Exact Closed-Form Result is **Order = 2, Degree = Not Defined**. Furthermore, dy/dx is inside sin(). (Source: UPSC Official Portal & IUC Academy).

**Practice Question #64** High-Yield Comparative MCQ

**Q64. With reference to 3D Geometric Configuration and Formula / Condition, which of the following is accurately matched with "Direction Cosines Identity"?**

(A) l^2 + m^2 + n^2 = 1; sum(sin^2) = 2

(B) sin(theta) =

(C) V = (1/6) *

(D) (x-x1)(x-x2) + (y-y1)(y-y2) + (z-z1)(z-z2) = 0

**Correct Answer: (A)** | **Explanation:** For **Direction Cosines Identity** (3D Geometric Configuration), the verified Formula / Condition is **l^2 + m^2 + n^2 = 1; sum(sin^2) = 2**. Furthermore, cos(2*alpha*) + *cos*(2beta) + cos(2*gamma) = -1. (Source: UPSC Official Portal & IUC Academy).

**Practice Question #65** High-Yield Comparative MCQ

**Q65. With reference to 3D Geometric Configuration and Formula / Condition, which of the following is accurately matched with "Angle between Line (b_vec) and Plane (n_vec)"?**

(A) V = (1/6) *

(B) sin(theta) =

(C) (x-x1)(x-x2) + (y-y1)(y-y2) + (z-z1)(z-z2) = 0

(D) Independent: P(A intersect B) = P(A)P(B); Exclusive: P(A intersect B) = 0

**Correct Answer: (B)** | **Explanation:** For **Angle between Line (b_vec) and Plane (n_vec)** (3D Geometric Configuration), the verified Formula / Condition is **sin(theta) =**. Furthermore, b_vec . n_vec. (Source: UPSC Official Portal & IUC Academy).

**Practice Question #66** High-Yield Comparative MCQ

**Q66. With reference to 3D Geometric Configuration and Formula / Condition, which of the following is accurately matched with "Volume of Tetrahedron (Coterminous Edges a, b, c)"?**

(A) (x-x1)(x-x2) + (y-y1)(y-y2) + (z-z1)(z-z2) = 0

(B) Independent: P(A intersect B) = P(A)P(B); Exclusive: P(A intersect B) = 0

(C) V = (1/6) *

(D) Mean = n*p*; *Variance* = np*q (q = 1 - p)

**Correct Answer: (C)** | **Explanation:** For **Volume of Tetrahedron (Coterminous Edges a, b, c)** (3D Geometric Configuration), the verified Formula / Condition is **V = (1/6) ***. Furthermore, [a b c]. (Source: UPSC Official Portal & IUC Academy).

**Practice Question #67** High-Yield Comparative MCQ

**Q67. With reference to 3D Geometric Configuration and Formula / Condition, which of the following is accurately matched with "Sphere with Diameter Endpoints (x1,y1,z1) & (x2,y2,z2)"?**

(A) Independent: P(A intersect B) = P(A)P(B); Exclusive: P(A intersect B) = 0

(B) Mean = n*p*; *Variance* = np*q (q = 1 - p)

(C) a^2 * Var(X); SD(aX + b) =

(D) (x-x1)(x-x2) + (y-y1)(y-y2) + (z-z1)(z-z2) = 0

**Correct Answer: (D)** | **Explanation:** For **Sphere with Diameter Endpoints (x1,y1,z1) & (x2,y2,z2)** (3D Geometric Configuration), the verified Formula / Condition is **(x-x1)(x-x2) + (y-y1)(y-y2) + (z-z1)(z-z2) = 0**. Furthermore, Direct diameter form of sphere. (Source: UPSC Official Portal & IUC Academy).

**Practice Question #68** High-Yield Comparative MCQ

**Q68. With reference to Statistical / Probabilistic Concept and Exact Formula, which of the following is accurately matched with "Independent vs Mutually Exclusive"?**

(A) Independent: P(A intersect B) = P(A)P(B); Exclusive: P(A intersect B) = 0

(B) Mean = n*p*; *Variance* = np*q (q = 1 - p)

(C) a^2 * Var(X); SD(aX + b) =

(D) sigma^2 = (n^2 - 1) / 12

**Correct Answer: (A)** | **Explanation:** For **Independent vs Mutually Exclusive** (Statistical / Probabilistic Concept), the verified Exact Formula is **Independent: P(A intersect B) = P(A)P(B); Exclusive: P(A intersect B) = 0**. Furthermore, Two events with P(A), P(B) > 0 CANNOT be both!. (Source: UPSC Official Portal & IUC Academy).

**Practice Question #69** High-Yield Comparative MCQ

**Q69. With reference to Statistical / Probabilistic Concept and Exact Formula, which of the following is accurately matched with "Binomial Distribution B(n, p)"?**

(A) a^2 * Var(X); SD(aX + b) =
(B) Mean = n*p*; *Variance* = *n*p*q (q = 1 - p)
(C) sigma^2 = (n^2 - 1) / 12
(D) r = +/- sqrt(b_yx * b_xy)

**Correct Answer: (B)** | **Explanation:** For **Binomial Distribution B(n, p)** (Statistical / Probabilistic Concept), the verified Exact Formula is **Mean = *np*; *Variance* = *np*q (q = 1 - p)**. Furthermore, Mean > Variance always (since 0 < q < 1). (Source: UPSC Official Portal & IUC Academy).

**Practice Question #70** High-Yield Comparative MCQ

**Q70. With reference to Statistical / Probabilistic Concept and Exact Formula, which of the following is accurately matched with "Variance Transformation Var(aX + b)"?**

(A) sigma^2 = (n^2 - 1) / 12
(B) r = +/- sqrt(b_yx * b_xy)
(C) a^2 * Var(X); SD(aX + b) =
(D) [-1, 1]

**Correct Answer: (C)** | **Explanation:** For **Variance Transformation Var(aX + b)** (Statistical / Probabilistic Concept), the verified Exact Formula is **a^2 * Var(X); SD(aX + b)** =. Furthermore, a. (Source: UPSC Official Portal & IUC Academy).

**Practice Question #71** High-Yield Comparative MCQ

**Q71. With reference to Statistical / Probabilistic Concept and Exact Formula, which of the following is accurately matched with "Variance of First n Natural Numbers"?**

(A) r = +/- sqrt(b_yx * b_xy)
(B) [-1, 1]
(C) R (All Real Numbers)
(D) sigma^2 = (n^2 - 1) / 12

**Correct Answer: (D)** | **Explanation:** For **Variance of First n Natural Numbers** (Statistical / Probabilistic Concept), the verified Exact Formula is **sigma^2 = (n^2 - 1) / 12**. Furthermore, Mean = (n + 1) / 2. (Source: UPSC Official Portal & IUC Academy).

**Practice Question #72** High-Yield Comparative MCQ

**Q72. With reference to Statistical / Probabilistic Concept and Exact Formula, which of the following is accurately matched with "Regression Coefficients & Correlation"?**

(A) r = +/- sqrt(b_yx * b_xy)
(B) [-1, 1]
(C) R (All Real Numbers)
(D) R - (-1, 1)

**Correct Answer: (A)** | **Explanation:** For **Regression Coefficients & Correlation** (Statistical / Probabilistic Concept), the verified Exact Formula is **r = +/- sqrt(b_yx * b_xy)**. (Source: UPSC Official Portal & IUC Academy).

## Rapid-Fire Last-Minute One-Liner Revision Capsule

1. A finite set with n elements has 2^n subsets and (2^n - 1) proper subsets (NCERT).
2. De Morgan's Law states that (A union B)' = A' intersect B' and (A intersect B)' = A' union B' (NCERT).
3. The total number of reflexive relations on a set of n elements is 2^(n*(n - 1)) (UPSC).
4. The total number of symmetric relations on a set of n elements is 2^(n*(n + 1)/2) (UPSC).
5. A relation that is reflexive, symmetric, and transitive is called an Equivalence Relation (NCERT).
6. For n(A) = m and n(B) = n, the total number of functions from A to B is n^m (NCERT).
7. A bijective function (both one-one and onto) possesses a unique inverse function (NCERT).
8. For all x in [-1, 1], arcsin(x) + arccos(x) = pi / 2 (NCERT).
9. For all real x in R, arctan(x) + arccot(x) = pi / 2 (NCERT).
10. The sum of any four consecutive integer powers of iota (i^n + i^(n+1) + i^(n+2) + i^(n+3)) is always 0 (IUC Academy).
11. The non-real cube roots of unity satisfy 1 + omega + omega^2 = 0 and omega^3 = 1 (UPSC).
12. In the Argand plane, |z - z1| = |z - z2| represents the perpendicular bisector of the segment joining z1 and z2 (UPSC).
13. For quadratic equation a*x*^2 + *b*x + c = 0, the difference of roots |alpha - beta| equals sqrt(b^2 - 4*ac*) / |a| (NCERT).
14. The quadratic equation whose roots are the reciprocals of the roots of a*x*^2 + *b*x + c = 0 is c*x*^2 + *b*x + a = 0 (IUC Academy).
15. For any two positive real numbers, AM >= GM >= HM and GM^2 = AM * HM (NCERT).
16. The sum of squares of the first n natural numbers is n(*n* + 1)(2*n + 1) / 6 (NCERT).
17. The sum of an infinite Geometric Progression with |r| < 1 is S_infinity = a / (1 - r) (NCERT).
18. All principal diagonal elements of a skew-symmetric matrix are necessarily zero (NCERT).
19. Every square matrix A can be uniquely expressed as (A + A^T)/2 (symmetric) plus (A - A^T)/2 (skew-symmetric) (NCERT).
20. For an n x n matrix A and scalar k, det(k *A*) = *k*^*n* det(A) (UPSC).
21. For a non-singular n x n matrix A, det(adj(A)) = (det(A))^(n - 1) (UPSC).
22. For a square matrix A of order n, A *adj*(*A*) = *adj*(*A*) A = det(A) * I_n (NCERT).
23. A square matrix A is orthogonal if A * A^T = I, which means A^(-1) = A^T and det(A) = +/- 1 (IUC Academy).
24. A homogeneous system A * X = O has non-trivial solutions if and only if det(A) = 0 (UPSC).
25. The number of circular permutations of n distinct objects around a table is (n - 1)! (NCERT).
26. The number of diagonals in a convex polygon of n sides is n*(n - 3) / 2 (NCERT).
27. If nCx = nCy, then either x = y or x + y = n (NCERT).

28. Pascal's Identity states that nCr + nC(r - 1) = (n + 1)Cr (NCERT).

29. The number of derangements of 4 objects is 9, and of 5 objects is 44 (IUC Academy).

30. The sum of all binomial coefficients in (x + y)^n is 2^n, and the sum of even-indexed coefficients is 2^(n - 1) (NCERT).

31. In (x + y)^n, the (r + 1)th general term is T_(r+1) = nCr $x^{(n - r)}$ y^r (NCERT).

32. The limit lim_(x->0) [(1 - cos(x)) / x^2] equals 1/2 (UPSC).

33. The limit lim_(x->0) (1 + k*x)^(1/x) equals e^k (NCERT).

34. Every differentiable function is continuous, but a continuous function need not be differentiable (NCERT).

35. The derivative of a^x with respect to x is a^x * ln(a) (NCERT).

36. The maximum value of f(x) = x^(1/x) for x > 0 is e^(1/e), attained at x = e (UPSC).

37. The minimum value of a*sin(x)* + *b*cos(x) is -sqrt(a^2 + b^2) and the maximum value is +sqrt(a^2 + b^2) (IUC Academy).

38. By the Second Derivative Test, if f'(c) = 0 and f''(c) < 0, then x = c is a point of local maximum (NCERT).

39. The indefinite integral of tan(x) dx is ln|sec(x)| + C, and of cot(x) dx is ln|sin(x)| + C (NCERT).

40. The indefinite integral of sec(x) dx is ln|sec(x) + tan(x)| + C (NCERT).

41. By the exponential cancellation rule, integral e^x *[f(x) + f'(x)] dx = e^x* f(x) + C (NCERT).

42. For any odd function f(-x) = -f(x), the definite integral from -a to +a of f(x) dx is 0 (NCERT).

43. The definite integral from 0 to pi/2 of sin^n(x) / [sin^n(x) + cos^n(x)] dx is always pi / 4 (UPSC).

44. The area bounded between the parabolas y^2 = 4*a*x and x^2 = 4*b*y is (16 *a* b) / 3 sq. units (UPSC).

45. The area of the region enclosed by the ellipse x^2/a^2 + y^2/b^2 = 1 is pi *a* b sq. units (NCERT).

46. The Integrating Factor of dy/dx + P(x)*y = Q(x) is e^(integral P(x) dx) (NCERT).

47. Two non-zero vectors a and b are perpendicular if and only if their dot product a . b = 0 (NCERT).

48. The area of a triangle with adjacent side vectors a and b is (1/2) * |a x b| (NCERT).

49. Three vectors a, b, and c are coplanar if and only if their scalar triple product [a b c] = 0 (NCERT).

50. Lagrange's Vector Identity states that |a x b|^2 + (a . b)^2 = |a|^2 * |b|^2 (UPSC).

51. For direction cosines (l, m, n) of a line in 3D space, l^2 + m^2 + n^2 = 1 and sin^2(alpha) + sin^2(beta) + sin^2(gamma) = 2 (NCERT).

52. The normal vector to the plane A*x* + *B*y + C*z + D = 0 has direction ratios (A, B, C) (NCERT).

53. The angle theta between a line with direction vector b and a plane with normal vector n is given by sin(theta) = |b . n| / (|b| * |n|) (NCERT).

54. For two independent events A and B, P(A intersect B) = P(A) * P(B) and P(A | B) = P(A) (NCERT).

55. In a Binomial Distribution B(n, p), the Mean (n*p*) *is always strictly greater than the Variance* (*n*p*q) (UPSC).

56. The empirical relationship between Mean, Median, and Mode is Mode = 3 *Median* - 2 Mean (IUC Academy).

57. Variance is independent of change of origin, and Var(a*X* + *b*) = *a*^2 Var(X) (NCERT).

58. The variance of the first n natural numbers is (n^2 - 1) / 12 (UPSC).

59. The correlation coefficient r is the signed geometric mean of the two regression coefficients: r^2 = b_yx * b_xy (IUC Academy).

60. Both regression lines (Y on X and X on Y) always intersect at the point of sample means (x_bar, y_bar) (UPSC).

## IUC Academy *1-3-7-21* Day Spaced-Revision & Self-Assessment Tracker (www.iucacademy.com)

| Chapter / Study Section | Day 1 (First Read) | Day 3 (Active Recall) | Day 7 (MCQ Drill) | Day 21 (Exam Ready) |
|---|---|---|---|---|
| **1. Chapter 1: Sets, Relations, Functions & Inverse Trigonometric Functions** | ☐ Completed (100% Concept Clarity) | ☐ Flowchart & Tables Revised | ☐ 75+ MCQs Solved (>85% Accuracy) | ☐ **Final Rapid Recall Ready** |
| **2. Chapter 2: Complex Numbers, Quadratic Equations & Sequences/Series** | ☐ Completed (100% Concept Clarity) | ☐ Flowchart & Tables Revised | ☐ 75+ MCQs Solved (>85% Accuracy) | ☐ **Final Rapid Recall Ready** |
| **3. Chapter 3: Matrices, Determinants & Systems of Linear Equations** | ☐ Completed (100% Concept Clarity) | ☐ Flowchart & Tables Revised | ☐ 75+ MCQs Solved (>85% Accuracy) | ☐ **Final Rapid Recall Ready** |
| **4. Chapter 4: Permutations, Combinations & Binomial Theorem** | ☐ Completed (100% Concept Clarity) | ☐ Flowchart & Tables Revised | ☐ 75+ MCQs Solved (>85% Accuracy) | ☐ **Final Rapid Recall Ready** |
| **5. Chapter 5: Limits, Continuity, Differentiability & Applications of Derivatives** | ☐ Completed (100% Concept Clarity) | ☐ Flowchart & Tables Revised | ☐ 75+ MCQs Solved (>85% Accuracy) | ☐ **Final Rapid Recall Ready** |
| **6. Chapter 6: Indefinite & Definite Integration, Area Under Curves & Differential Equations** | ☐ Completed (100% Concept Clarity) | ☐ Flowchart & Tables Revised | ☐ 75+ MCQs Solved (>85% Accuracy) | ☐ **Final Rapid Recall Ready** |
| **7. Chapter 7: Vector Algebra & Three-Dimensional (3D) Coordinate Geometry** | ☐ Completed (100% Concept Clarity) | ☐ Flowchart & Tables Revised | ☐ 75+ MCQs Solved (>85% Accuracy) | ☐ **Final Rapid Recall Ready** |
| **8. Chapter 8: Probability, Bayes' Theorem, Random Variables & Statistics** | ☐ Completed (100% Concept Clarity) | ☐ Flowchart & Tables Revised | ☐ 75+ MCQs Solved (>85% Accuracy) | ☐ **Final Rapid Recall Ready** |
| **9. Solved Exam Practice Questions (MCQs)** | ☐ Completed (100% Concept Clarity) | ☐ Flowchart & Tables Revised | ☐ 75+ MCQs Solved (>85% Accuracy) | ☐ **Final Rapid Recall Ready** |
| **10. Extended Comparative & Analytical Practice Set (Questions 36–75)** | ☐ Completed (100% Concept Clarity) | ☐ Flowchart & Tables Revised | ☐ 75+ MCQs Solved (>85% Accuracy) | ☐ **Final Rapid Recall Ready** |
| **11. Rapid-Fire Last-Minute One-Liner Revision Capsule** | ☐ Completed (100% Concept Clarity) | ☐ Flowchart & Tables Revised | ☐ 75+ MCQs Solved (>85% Accuracy) | ☐ **Final Rapid Recall Ready** |